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# A General Class of Product-cum-Ratio-Type Exponential Estimators in Double Sampling for Stratification of Finite Population Mean

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## Abstract

This paper addresses the problem of estimating the population mean of the study variable $y$ using information on the auxiliary variable $x$ in double sampling for stratification. A general class of product-cum-ratio-type estimators have been defined in this paper. The properties of the suggested class of estimators are studied up to terms of order $O\left(n^{-1}\right)$. Asymptotic optimum estimator (*AOE*) in the class is also identified. In particular, to illustrate the general results, we have suggested a subclass of estimators $\hat{\bar{Y}}_{(1)}$ along with its properties. Preference regions are obtained in which the proposed estimator $\hat{\bar{Y}}_{(1)}$ is better than the existing estimators. In support of the present study, an empirical study is also carried out.

*Key words*: Auxiliary variable; Bias, Mean squared error; Double sampling for stratification.

**Mathematics Subject Classification Code:** 62D05.

## 1. Introduction

It is a well-established fact that the use of auxiliary information in the estimation of population mean provides efficient estimators. Out of many, ratio, product and regression methods of estimation are good examples in this context. A large amount of work has been carried out in estimating the population mean using simple random sampling (*SRS*) with or without replacement (*WOR*) scheme, for instance, see Singh (1986), Singh (2003) among others. Usually, heterogeneous populations are encountered in practice. In such a situation, stratification (or stratified sampling) is extensively used procedure in sample surveys to provide samples that are representatives of major sub-groups of a population. When the sampling frame within strata is known, stratified sampling is used, but there are many situations of practical importance where strata weights are known but a frame within the strata is not available; post-stratification may then be employed to cope with this problem. For example, in household survey in a city, number of households in different colonies may be available, but list of households may not be available. In such a situation post-stratification is used. However, in other situations with the passage of time, the stratum weights may not be known exactly as they become out-of-date. Further, the information on the stratification variable may not be readily available but could be made available by diverting a part of the survey budget to its collection. This type of situation occurs during the household surveys, when the investigator does not have information about newly added households in different colonies, see Tailor *et al.*

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(2014). In such a situation we employ the procedure of double sampling for stratification (*DSS*) introduced by Neyman (1938). Double sampling for stratification is a sampling design that is extensively employed in forest and other resource inventories in forest ecosystems. Double sampling is a powerful and cost-effective procedure. For more studies on this topic the reader is referred to the papers by Rao (1973), Ige and Tripathi (1987, 1991), Singh and Vishwakarma (2007), Vishwakarma and Singh (2012), Tailor and Lone (2014), Vishwakarma and Zeeshan (2018) and Singh and Nigam (2020 a, b).

## 2. Procedure of Selecting a Sample and Terminologies

Consider a finite population $U = \{U_1, U_2, ..U_N\}$ of $N$ units. Let $y$ and $x$ be study variate and auxiliary variate, respectively. Let $\bar{Y}$ be the population mean of the study variate y. Suppose we want to estimate the population mean $\bar{Y}$ of $y$ and consider it desirable to stratify the population on the basis of the values of ran auxiliary variate $x$ but the frequency distribution of $x$ is unknown. Let the population of size $N$ be stratified into $L$ strata of size $N_h$ with strata weights $W_h = \frac{N_h}{N}$, ($h$ = 1,2,…,$L$). The sampling frame for different strata and the strata weights $W_h = \frac{N_h}{N}$, $h$ = 1,2,…,$L$ are not known although the strata may be fixed in advance. Under these circumstances we employ the procedure of double sampling for stratification (*DSS*). It consists of the following steps [see Rao (1973) and Ige and Tripathi (1987)]:

(i) We select first phase sample $S'$ of size $n'$ using simple random sampling without replacement (*SRSWOR*) and measure only auxiliary variate $x$.

(ii) The first phase sample $S'$ is stratified into $L$ strata based on measured $x$-values. Let $n'_h$ be the number of units in $S'$ falling into stratum $h$ ($h$=1,2,…,$L$; $n' = \sum_{h=1}^{L} n'_h$ ) and $n' = \{n'_1, n'_2, ..., n'_L\}$ denote the resulting configuration of $S'$.

(iii) Sub-sample of sizes $n_h = \upsilon_h n'_h, 0 < \upsilon_h < 1, h = 1,2,...,L$, $\upsilon_h$ being predetermined for all $h$, are drawn from strata, independently from each other, using *SRSWOR*. Thus, it constitutes a second phase sample $S$ of size $n = \sum_{h=1}^{L} n_h$ . The study variable $y$ is measured on all $n_h$ sampled unites, for all $h$.

We use the following notations:

$w_h = \frac{n'_h}{n'}$ :is an unbiased estimator of strata weights $W_h = \frac{N_h}{N}$ (or proportion of first sample falling in stratum $h$), see Cochran (1977, p.328),

$\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$ is the population mean of the study variable $y$,

$\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$ is the population mean of the auxiliary variable $x$.

$S_y^2 = \frac{1}{N-1} \sum_{i=1}^{N} (y_i - \bar{Y})^2$ is the population mean square of $y$,

$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2$ is the population mean square of $x$,

$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_i - \bar{X}\right)$ is the population covariance between $y$ and $x$,

$S_{yh}^2 = \frac{1}{N_h - 1}\sum_{j=1}^{N_h}\left(y_{hj} - \bar{Y}_h\right)^2$ is the population mean square of $y$ of $h^{th}$ stratum,

$S_{xh}^2 = \frac{1}{N_h - 1}\sum_{j=1}^{N_h}\left(x_{hj} - \bar{X}_h\right)^2$ is the population mean square of $x$ of $h^{th}$ stratum,

$\rho_{yxh} = \frac{S_{yxh}}{S_{yh}S_{xh}}$ is the population correlation coefficient between $y$ and $x$ in the $h^{th}$ stratum,

$\bar{y}_{ds} = \sum_{h=1}^{L} w_h \bar{y}_h$ is an unbiased estimator of the population mean $\bar{Y}$,

$\bar{x}_{ds} = \sum_{h=1}^{L} w_h \bar{x}_h$ is an unbiased estimator of the population mean $\bar{X}$,

$\bar{x}' = \sum_{h=1}^{L} w_h \bar{x}'_h$ is an unbiased estimator of the population mean $\bar{X}$,

$\bar{y}_h = \frac{1}{n_h}\sum_{j=1}^{n_h} y_{hj}$ is the mean of the second phase taken from $h^{th}$ stratum for $y$,

$\bar{x}_h = \frac{1}{n_h}\sum_{j=1}^{n_h} x_{hj}$ is the mean of the second phase taken from $h^{th}$ stratum for $x$,

$\bar{x}'_h = \frac{1}{n'_h}\sum_{j=1}^{n'_h} x_{hj}$ is the mean of the first phase sample of $h^{th}$ stratum for $x$,

$f = \frac{n'}{N}$ is the first phase sampling fraction,

$\beta_h = \frac{S_{yxh}}{S_{xh}^2}, a_h = \left(\frac{1}{\upsilon_h} - 1\right)W_h S_{xh}^2, \beta = \frac{\sum_{h=1}^{L} a_h \beta_h}{\sum_{h=1}^{L} a_h} = \frac{A_{yx}}{A_x}$ is the weighted average of the strata population regression coefficient, $R = \frac{\bar{Y}}{\bar{X}}$,

$$A_y = \sum_{h=1}^{L} W_h S_{yh}^2 \left(\frac{1}{\upsilon_h} - 1\right),$$

$$A_x = \sum_{h=1}^{L} W_h S_{xh}^2 \left(\frac{1}{\upsilon_h} - 1\right),$$

$$A_{yx} = \sum_{h=1}^{L} W_h S_{yxh} \left(\frac{1}{\upsilon_h} - 1\right) \text{and } \rho = \frac{A_{yx}}{\sqrt{A_y A_x}}.$$

To obtain the bias and mean squared error (*MSE*) of the suggested estimator, we write

$$\bar{y}_{ds} = \bar{Y}(1+e_0), \bar{x}_{ds} = \bar{X}(1+e_1) \text{and } \bar{x}' = \bar{X}(1+e'_1)$$

such that

$$E(e_0) = E(e_1) = E(e'_1) = 0$$

and

$$E(e_0^2) = \frac{1}{\bar{Y}^2}\left[S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}A_y\right],$$

$$E(e_1^2) = \frac{1}{\bar{X}^2}\left[S_x^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}A_x\right],$$

$$E(e'^2_1) = \left(\frac{1-f}{n'}\right)\frac{1}{\bar{X}^2}S_x^2,$$

$$E(e_0 e_1) = \frac{1}{\bar{Y}\,\bar{X}}\left[S_{yx}\left(\frac{1-f}{n'}\right) + \frac{1}{n'}A_{yx}\right],$$

$$E(e_0 e'_1) = \left(\frac{1-f}{n'}\right)\frac{1}{\bar{Y}\,\bar{X}}S_{yx}, \text{ and}$$

$$E(e_1 e'_1) = \left(\frac{1-f}{n'}\right)\frac{1}{\bar{X}^2}S_x^2.$$

## 3. Review of Some Existing Estimators

The usual unbiased estimator for population mean $\bar{Y}$ is defined by

$$\bar{y}_{ds} = \sum_{h=1}^{L} w_h \bar{y}_h. \tag{1}$$

The variance/*MSE* of $\bar{y}_{ds}$ is given by

$$V(\bar{y}_{ds}) = MSE(\bar{y}_{ds}) = \left(\frac{1-f}{n'}\right)S_y^2 + \frac{1}{n'}A_y. \tag{2}$$

In double sampling for stratification, ratio and product estimators due to Ige and Tripathi (1987) are respectively given by

$$\hat{\bar{Y}}_{R(ds)} = \bar{y}_{ds}\left(\frac{\bar{x}'}{\bar{x}_{ds}}\right), \tag{3}$$

$$\hat{\bar{Y}}_{P(ds)} = \bar{y}_{ds}\left(\frac{\bar{x}_{ds}}{\bar{x}'}\right). \tag{4}$$

The *MSE*s of $\hat{\bar{Y}}_{R(ds)}$ and $\hat{\bar{Y}}_{P(ds)}$ up to terms of order $O(n^{-1})$, are given respectively by

$$MSE(\hat{\bar{Y}}_{R(ds)}) = \left(\frac{1-f}{n'}\right)S_y^2 + \frac{1}{n'}\left\{A_y + R^2 A_x\left(1 - \frac{2\beta}{R}\right)\right\}, \tag{5}$$

$$MSE\left(\hat{\bar{Y}}_{P(ds)}\right)=\left(\frac{1-f}{n'}\right)S_y^2+\frac{1}{n'}\left\{A_y+R^2A_x\left(1+\frac{2\beta}{R}\right)\right\}, \tag{6}$$

where $\upsilon_h=\dfrac{n_h}{n'_h}$.

Motivated by Bahl and Tuteja (1991), Tailor *et al*. (2014) suggested the ratio-type and product-type exponential estimators in double sampling for stratification for population mean $\bar{Y}$ respectively as

$$\hat{\bar{Y}}_{\mathrm{Re}(ds)}=\bar{y}_{ds}\exp\left\{\frac{\bar{x}'-\bar{x}_{ds}}{\bar{x}'+\bar{x}_{ds}}\right\}, \tag{7}$$

$$\hat{\bar{Y}}_{Pe(ds)}=\bar{y}_{ds}\exp\left\{\frac{\bar{x}_{ds}-\bar{x}'}{\bar{x}'+\bar{x}_{ds}}\right\}. \tag{8}$$

The *MSE*s of $\hat{\bar{Y}}_{\mathrm{Re}(ds)}$ and $\hat{\bar{Y}}_{Pe(ds)}$ up to terms of order $O\left(n^{-1}\right)$, are respectively given by

$$MSE\left(\hat{\bar{Y}}_{\mathrm{Re}(ds)}\right)=\left(\frac{1-f}{n'}\right)S_y^2+\frac{1}{n'}\left\{A_y+\frac{R^2A_x}{4}\left(1-\frac{4\beta}{R}\right)\right\}, \tag{9}$$

$$MSE\left(\hat{\bar{Y}}_{Pe(ds)}\right)=\left(\frac{1-f}{n'}\right)S_y^2+\frac{1}{n'}\left\{A_y+\frac{R^2A_x}{4}\left(1+\frac{4\beta}{R}\right)\right\}. \tag{10}$$

From (2), (5), (6), (9) and (10) it can be observed that the:

(i) ratio estimator $\hat{\bar{Y}}_{R(ds)}$ is more efficient than the unbiased estimator $\bar{y}_{(ds)}$ if

$$\frac{\beta}{R}>\frac{1}{2} \tag{11}$$

(ii) product estimator $\hat{\bar{Y}}_{P(ds)}$ is better than the unbiased estimator $\bar{y}_{(ds)}$ if

$$\frac{\beta}{R}<-\frac{1}{2} \tag{12}$$

(iii) ratio-type exponential estimator $\hat{\bar{Y}}_{\mathrm{Re}(ds)}$ is superior to the unbiased estimator $\bar{y}_{(ds)}$ if

$$\frac{\beta}{R}>\frac{1}{4} \tag{13}$$

(iv) product-type exponential estimator $\hat{\bar{Y}}_{Pe(ds)}$ is more precise than the unbiased estimator $\bar{y}_{(ds)}$ if

$$\frac{\beta}{R} < -\frac{1}{4} \tag{14}$$

In this paper we have suggested a general class of product-cum-ratio-type exponential estimators for population mean $\bar{Y}$ of $y$ using double sampling for stratification based on auxiliary information. Expressions of bias and *MSE* of the proposed estimator up to $O(n^{-1})$ are derived. Asymptotic optimum estimator (*AOE*) in the class is identified with its approximate *MSE* formula. To illustrate the general results we have considered a subclass of estimators $\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)}$ along with its properties up to terms of order $O(n^{-1})$. An empirical study is carried out in support of the present study.

## 4. The Suggested Class of Product-Cum-Ratio-Type Exponential Estimators

We define a general class of product-cum-ratio-type exponential estimators for population mean $\bar{Y}$ in double sampling for stratification based on auxiliary information, as

$$\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)} = \bar{y}_{ds}\left[\delta\left(\frac{a\bar{x}_{ds}+b}{a\bar{x}'+b}\right)^{\eta} + (1-\delta)\exp\left\{\frac{\phi a(\bar{x}'-\bar{x}_{ds})}{a(\bar{x}'+\bar{x}_{ds})+2b}\right\}\right], \tag{15}$$

where $\delta$ is a suitable chosen constant, $(\eta,\phi)$ are scalars taking values (-1,0,1) for generating ratio and product-type estimators, $a\,(\neq 0)$ and $b$ are either real numbers or functions of known parameters of the auxiliary variable $x$ like coefficient of variation $C_x$, standard deviation $S_x$, coefficient of skewness $\beta_1(x)$, coefficient of kurtosis $\beta_2(x)$, correlation coefficient $\rho_{yx}$ between $y$ and $x$; and $\Delta(x) = (\beta_2(x) - \beta_1(x) - 1)$. One may also take the values of $a$ and $b$ as $\varphi_1 = \sum_{h=1}^{L} W_h S_{xh}, \varphi_2 = \sum_{h=1}^{L} W_h C_{xh}, \varphi_3 = \sum_{h=1}^{L} W_h \beta_{1h}(x), \varphi_4 = \sum_{h=1}^{L} W_h \beta_{2h}(x), \varphi_5 = \sum_{h=1}^{L} W_h \rho_{yxh}$ and $\varphi_6 = \sum_{h=1}^{L} W_h \Delta_h(x)$ with $\Delta_h(x) = (\beta_{2h}(x) - \beta_{1h}(x) - 1)$, for instance, see Koyuncu and Kadilar (2009, p.2553).

A large number of estimators can be generated from the proposed estimator $\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)}$ for suitable values of $(\delta,\eta,a,b)$. For example:

(i) $\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)} \to \bar{y}_{ds}$ for $(a,b,\delta,\eta,\phi) = (a,b,\delta,0,0)$,

(ii) $\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)} \to \hat{\bar{Y}}_{R(ds)}$ for $(a,b,\delta,\eta,\phi) = (a,0,1,-1,\phi)$,

(iii) $\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)} \to \hat{\bar{Y}}_{P(ds)}$ for $(a,b,\delta,\eta,\phi) = (a,0,1,1,\phi)$,

(iv) $\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)} \to \hat{\bar{Y}}_{\mathrm{Re}(ds)}$ for $(a,b,\delta,\eta,\phi) = (a,0,0,\eta,1)$,

(v) $\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)} \to \hat{\bar{Y}}_{Pe(ds)}$ for $(a,b,\delta,\eta,\phi) = (a,0,0,\eta,-1)$,

*etc*.

### 4.1. The bias and *MSE* of the proposed class of estimators $\hat{\hat{Y}}^{(c)}_{P\mathrm{Re}(ds)}$

Expressing $\hat{\hat{Y}}^{(c)}_{P\mathrm{Re}(ds)}$ at (15) in terms of $e$'s we have

$$\hat{\hat{Y}}^{(c)}_{P\mathrm{Re}(ds)} = \bar{Y}\left(1+e_0\right)\left[\delta\left(\frac{a\bar{X}\left(1+e_1\right)+b}{a\bar{X}\left(1+e'_1\right)+b}\right)^{\eta} + (1-\delta)\exp\left\{\frac{\phi a\bar{X}\left(e'_1-e_1\right)}{a\bar{X}\left(2+e'_1+e_1\right)+2b}\right\}\right]$$

$$= \bar{Y}\left(1+e_0\right)\left[\delta\left(1+\xi e_1\right)^{\eta}\left(1+\xi e'_1\right)^{-\eta} + (1-\delta)\exp\left\{\frac{\xi\phi}{2}\left(e'_1-e_1\right)\left(1+\frac{\xi}{2}\left(e'_1+e_1\right)\right)^{-1}\right\}\right], \quad (16)$$

where the $\xi = \dfrac{a\bar{X}}{\left(a\bar{X}+b\right)}$.

Expanding right hand side of (16), multiplying out, subtracting $\bar{Y}$ from both sides and neglecting terms of $e$'s having power greater than two we have

$$\left(\hat{\hat{Y}}^{(c)}_{P\mathrm{Re}(ds)} - \bar{Y}\right) \cong \bar{Y}\left[\begin{array}{l} e_0 + \frac{1}{2}\{2\eta\delta-(1-\delta)\phi\}\xi\left(e_1-e'_1+e_0e_1-e_0e'_1\right) + \frac{\xi^2}{8}\{4\delta\eta(\eta-1)+(1-\delta)\phi(\phi+2)\}e_1^2 \\ +\frac{\xi^2}{8}\{4\delta\eta(\eta+1)+(1-\delta)\phi(\phi-2)\}e'^2_1 - \frac{\xi^2}{4}\{4\delta\eta^2+(1-\delta)\phi^2\}e_1e'_1 \end{array}\right] \quad (17)$$

Taking expectation of both sides of (17) we get the bias of $\hat{\hat{Y}}^{(c)}_{P\mathrm{Re}(ds)}$ up to terms of order $O\left(n^{-1}\right)$ as

$$B\left(\hat{\hat{Y}}^{(c)}_{P\mathrm{Re}(ds)}\right) = \frac{\xi}{8n'\bar{X}}\left[4\{2\delta\eta-(1-\delta)\phi\}A_{yx} + \xi R\{4\delta\eta(\eta-1)+(1-\delta)\phi(\phi+2)\}A_x\right]. \quad (18)$$

Squaring both sides of (17), neglecting terms of $e$'s having power greater than two and then taking expectation of both sides we get the *MSE* of $\hat{\hat{Y}}^{(c)}_{P\mathrm{Re}(ds)}$ up to terms of order $O\left(n^{-1}\right)$ as

$$MSE\left(\hat{\hat{Y}}^{(c)}_{P\mathrm{Re}(ds)}\right) = \left[\frac{(1-f)}{n'}S_y^2 + \frac{1}{n'}\left\{A_y + \frac{\theta R^2\xi^2 A_x}{4}\left(\theta - \frac{4\beta}{R\xi}\right)\right\}\right], \quad (19)$$

which is minimized for

$$\theta = \frac{2\beta}{R\xi} = \theta_0 \text{(say)}, \quad (20)$$

where $\theta = \{(1-\delta)\phi - 2\delta\eta\}$.

Thus the resulting minimum *MSE* of $\hat{\hat{Y}}^{(c)}_{P\mathrm{Re}(ds)}$ up to terms of order $O\left(n^{-1}\right)$ is given by

$$MSE_{\min}\left(\hat{\hat{Y}}^{(c)}_{P\text{Re}(ds)}\right)=\left[\frac{(1-f)}{n'}S_y^2+\frac{A_y}{n'}\left(1-\rho^2\right)\right]. \tag{21}$$

Now we state the following theorem.

**Theorem 1:** Up to terms of order $O\left(n^{-1}\right)$,

$$MSE\left(\hat{\hat{Y}}^{(c)}_{P\text{Re}(ds)}\right)\geq\left[\frac{(1-f)}{n'}S_y^2+\frac{A_y}{n'}\left(1-\rho^2\right)\right]$$

with equality holding if

$$\theta=\frac{2\beta}{\xi R}.$$

To illustrate the general results of the class of estimators $\hat{\hat{Y}}^{(c)}_{P\text{Re}(ds)}$, we consider the following sub class of estimators for $\bar{Y}$ as

$$\hat{\hat{\bar{Y}}}_{(1)}=\bar{y}_{ds}\left[\delta\left(\frac{a\bar{x}_{ds}+b}{a\bar{x}'+b}\right)+(1-\delta)\exp\left\{\frac{a(\bar{x}'-\bar{x}_{ds})}{a(\bar{x}'+\bar{x}_{ds})+2b}\right\}\right] \tag{22}$$

which is obtained on putting $(\eta,\phi)=(1,1)$ in (15). We designate the estimator $\hat{\hat{\bar{Y}}}_{(1)}$ as 'product-cum-ratio-type exponential' estimator.

Inserting $(\eta,\phi)=(1,1)\Rightarrow\theta=(1-3\delta)$ in (18) and (19) we get the bias and *MSE* of $\hat{\hat{\bar{Y}}}_{(1)}$ up to terms of order $O\left(n^{-1}\right)$ respectively as

$$B\left(\hat{\hat{\bar{Y}}}_{(1)}\right)=\frac{\xi^2 RA_x}{2n'\bar{X}}\left[\frac{(3\delta-1)\beta}{\xi R}+\frac{3}{4}(1-\delta)\right], \tag{23}$$

$$MSE\left(\hat{\hat{\bar{Y}}}_{(1)}\right)=\left[\frac{(1-f)}{n'}S_y^2+\frac{1}{n'}\left\{A_y+\frac{(3\delta-1)R^2\xi^2A_x}{4}\left((3\delta-1)+\frac{4\beta}{R\xi}\right)\right\}\right]. \tag{24}$$

The $MSE\left(\hat{\hat{\bar{Y}}}_{(1)}\right)$ at (24) is minimum when

$$\delta=\frac{1}{3}\left(1-\frac{2\beta}{R\xi}\right)=\delta_{(o)}\text{ (say)}. \tag{25}$$

Substitution of (25) in (24) yields the minimum *MSE* of $\hat{\hat{\bar{Y}}}_{(1)}$ as

$$MSE_{\min}\left(\hat{\bar{Y}}_{(1)}\right)=\left[\frac{(1-f)}{n'}S_y^2+\frac{A_y}{n'}\left(1-\rho^2\right)\right]. \tag{26}$$

Now we give the conditions under which the proposed estimator $\hat{\bar{Y}}_{(1)}$ is more efficient than the estimators $\bar{y}_{ds}$, $\hat{\bar{Y}}_{R(ds)}$, $\hat{\bar{Y}}_{P(ds)}$, $\hat{\bar{Y}}_{\text{Re}(ds)}$ and $\hat{\bar{Y}}_{Pe(ds)}$.

From (2), (5), (6), (9), (10) and (24) it can be easily shown that the proposed product-cum-ratio-type exponential estimator $\hat{\bar{Y}}_{(1)}$ is more efficient than:

(i) the usual unbiased estimator $\bar{y}_{ds}$ if

$$\min\left\{\frac{1}{3},-\frac{1}{3}\left(\frac{4\beta}{R\xi}-1\right)\right\}<\delta<\max\left\{\frac{1}{3},-\frac{1}{3}\left(\frac{4\beta}{R\xi}-1\right)\right\}. \tag{27}$$

(ii) the ratio estimator $\hat{\bar{Y}}_{R(ds)}$ if

$$\min\left\{\frac{1}{3}\left(1-\frac{2}{\xi}\left(\frac{2\beta}{R}-1\right)\right),\frac{1}{3}\left(1-\frac{2}{\xi}\right)\right\}<\delta<\max\left\{\frac{1}{3}\left(1-\frac{2}{\xi}\left(\frac{2\beta}{R}-1\right)\right),\frac{1}{3}\left(1-\frac{2}{\xi}\right)\right\}. \tag{28}$$

(iii) the product estimator $\hat{\bar{Y}}_{P(ds)}$ if

$$\min\left\{\frac{1}{3}\left(1-\frac{2}{\xi}\left(\frac{2\beta}{R}+1\right)\right),\frac{1}{3}\left(1+\frac{2}{\xi}\right)\right\}<\delta<\max\left\{\frac{1}{3}\left(1-\frac{2}{\xi}\left(\frac{2\beta}{R}+1\right)\right),\frac{1}{3}\left(1+\frac{2}{\xi}\right)\right\}. \tag{29}$$

(iv) the ratio-type exponential estimator $\hat{\bar{Y}}_{\text{Re}(ds)}$ if

$$\min\left\{\frac{1}{3}\left(1-\frac{1}{\xi}\right),\frac{1}{3}\left(\frac{1}{\xi}\left(1-\frac{4\beta}{R}\right)+1\right)\right\}<\delta<\max\left\{\frac{1}{3}\left(1-\frac{1}{\xi}\right),\frac{1}{3}\left(\frac{1}{\xi}\left(1-\frac{4\beta}{R}\right)+1\right)\right\}. \tag{30}$$

(v) the product-type exponential estimator $\hat{\bar{Y}}_{Pe(ds)}$ if

$$\min\left\{\frac{1}{3}\left(1-\frac{1}{\xi}\left(\frac{4\beta}{R}+1\right)\right),\frac{1}{3}\left(\frac{1}{\xi}+1\right)\right\}<\delta<\max\left\{\frac{1}{3}\left(1-\frac{1}{\xi}\left(\frac{4\beta}{R}+1\right)\right),\frac{1}{3}\left(\frac{1}{\xi}+1\right)\right\}. \tag{31}$$

## 5. Numerical Illustration

To illustrate the performance of the suggested estimator $\hat{\bar{Y}}_{(1)}$ over other existing estimators, we have considered three data sets whose descriptions are given below.

**Data 1 [Source: Tailor *et al*. (2014)]**
$x$: Area in '000 Hectare, $y$: Productivity (MT/Hectare)
$N$=20, $n$=8, $n_1=4, n_2=4, n'_1=7, n'_2=7, N_1=10, N_2=10, \bar{Y}_1=142.80, \bar{Y}_2=102.60,$
$\bar{X}_1=1632.00, \bar{X}_2=2036.00, S_{x1}=102.17, S_{x2}=103.46, S_{y1}=6.09, S_{y2}=12.60,$
$S_{yx1}=-239.30, S_{yx2}=-655.30, S_y^2=528.43.$

**Data 2 [Source: Chouhan (2012)]**
$x$: Area in '000 Hectare, $y$: Productivity (MT/Hectare)
$N$=20, $n$=8, $n_1=4, n_2=4, n'_1=7, n'_2=7, N_1=10, N_2=10, \bar{Y}_1=1.70, \bar{Y}_2=3.67, \bar{X}_1=6.32,$
$\bar{X}_2=8.67, S_{x1}=1.19, S_{x2}=10.82, S_{y1}=0.50, S_{y2}=1.41, S_{yx1}=-0.05, S_{yx2}=-7.04, S_y^2=2.20.$

**Data 3 [Source: Murthy (1967), p228]**
$x$: Fixed capital, $y$: Output
$N$=10, $n$=4, $n_1=2, n_2=2, n'_1=4, n'_2=4, N_1=5, N_2=5, \bar{Y}_1=1925.8, \bar{Y}_2=3115.6,$
$\bar{X}_1=214.4, \bar{X}_2=333.8, S_{x1}=74.87, S_{x2}=66.35, S_{y1}=615.92, S_{y2}=340.38,$
$S_{yx1}=39360.68, S_{yx2}=22356.50, S_y^2=668351.00.$

We have computed the percent relative efficiencies ($PRE$s) of the suggested estimator $\hat{\hat{\bar{Y}}}_{(1)}$ with respect to $\bar{y}_{(ds)}$, $\hat{\hat{\bar{Y}}}_{R(ds)}$, $\hat{\hat{\bar{Y}}}_{P(ds)}$, $\hat{\hat{\bar{Y}}}_{\mathrm{Re}(ds)}$ and $\hat{\hat{\bar{Y}}}_{Pe(ds)}$ by using the following formulae:

$$PRE\left(\hat{\hat{\bar{Y}}}_{(1)}, \bar{y}_{ds}\right)=\frac{\left[\left(\frac{1-f}{n'}\right)S_y^2+\frac{1}{n'}A_y\right]}{\left[\frac{(1-f)}{n'}S_y^2+\frac{1}{n'}\left\{A_y+\frac{(3\delta-1)R^2\xi^2A_x}{4}\left((3\delta-1)+\frac{4\beta}{R\xi}\right)\right\}\right]}*100, \tag{32}$$

$$PRE\left(\hat{\hat{\bar{Y}}}_{(1)}, \hat{\hat{\bar{Y}}}_{R(ds)}\right)=\frac{\left[\left(\frac{1-f}{n'}\right)S_y^2+\frac{1}{n'}\left\{A_y+R^2A_x\left(1-\frac{2\beta}{R}\right)\right\}\right]}{\left[\frac{(1-f)}{n'}S_y^2+\frac{1}{n'}\left\{A_y+\frac{(3\delta-1)R^2\xi^2A_x}{4}\left((3\delta-1)+\frac{4\beta}{R\xi}\right)\right\}\right]}*100, \tag{33}$$

$$PRE\left(\hat{\hat{\bar{Y}}}_{(1)}, \hat{\hat{\bar{Y}}}_{P(ds)}\right)=\frac{\left[\left(\frac{1-f}{n'}\right)S_y^2+\frac{1}{n'}\left\{A_y+R^2A_x\left(1+\frac{2\beta}{R}\right)\right\}\right]}{\left[\frac{(1-f)}{n'}S_y^2+\frac{1}{n'}\left\{A_y+\frac{(3\delta-1)R^2\xi^2A_x}{4}\left((3\delta-1)+\frac{4\beta}{R\xi}\right)\right\}\right]}*100, \tag{34}$$

$$PRE\left(\hat{\hat{\bar{Y}}}_{(1)}, \hat{\hat{\bar{Y}}}_{\mathrm{Re}(ds)}\right)=\frac{\left[\left(\frac{1-f}{n'}\right)S_y^2+\frac{1}{n'}\left\{A_y+\frac{R^2A_x}{4}\left(1-\frac{4\beta}{R}\right)\right\}\right]}{\left[\frac{(1-f)}{n'}S_y^2+\frac{1}{n'}\left\{A_y+\frac{(3\delta-1)R^2\xi^2A_x}{4}\left((3\delta-1)+\frac{4\beta}{R\xi}\right)\right\}\right]}*100, \tag{35}$$

$$PRE\left(\hat{\hat{Y}}_{(1)}, \hat{\hat{Y}}_{Pe(ds)}\right) = \frac{\left[\left(\frac{1-f}{n'}\right)S_y^2 + \frac{1}{n'}\left\{A_y + \frac{R^2 A_x}{4}\left(1+\frac{4\beta}{R}\right)\right\}\right]}{\left[\frac{(1-f)}{n'}S_y^2 + \frac{1}{n'}\left\{A_y + \frac{(3\delta-1)R^2\xi^2 A_x}{4}\left((3\delta-1)+\frac{4\beta}{R\xi}\right)\right\}\right]} * 100. \quad (36)$$

Here we note that for the sake of simplicity we have taken (*a*, *b*) = (1, 0) for computing the *PRE* in the suggested estimator $\hat{\hat{Y}}_{(1)}$. Also for computing the range of $\delta$ we have taken (*a*, *b*) = (1, 0) in equation (27)-(31).

**Table 1: Different ranges of $\delta$ in which the proposed estimator $\hat{\hat{Y}}_{(1)}$ is more efficient than $\bar{y}_{(ds)}$, $\hat{\hat{Y}}_{R(ds)}$, $\hat{\hat{Y}}_{P(ds)}$, $\hat{\hat{Y}}_{\text{Re}(ds)}$ and $\hat{\hat{Y}}_{Pe(ds)}$**

| Estimators | Data 1 | | Data 2 | | Data 3 | |
|---|---|---|---|---|---|---|
| | $\delta$ (min) | $\delta$ (max) | $\delta$ (min) | $\delta$ (max) | $\delta$ (min) | $\delta$ (max) |
| $\bar{y}_{ds}$ | 0.33333 | 1.17659 | 0.33333 | 0.55604 | 0.33333 | –0.5608 |
| $\hat{\hat{Y}}_{R(ds)}$ | –0.33333 | 1.84326 | –0.33333 | 1.22271 | –0.33333 | 0.10589 |
| $\hat{\hat{Y}}_{P(ds)}$ | 0.50993 | 1 | –0.11063 | 1 | –1.22745 | 1 |
| $\hat{\hat{Y}}_{RExp(ds)}$ | 0 | 1.50993 | 0 | 0.88937 | –0.2274 | 0 |
| $\hat{\hat{Y}}_{PExp(ds)}$ | 0.84326 | 0.66667 | 0.22271 | 0.66667 | –0.89411 | 0.66667 |
| $\delta_{(opt)}$ | 0.75946 | | 0.44469 | | –0.1137 | |

**Table 2: *PRE* of $\hat{\hat{Y}}_{(1)}$ with respect to $\bar{y}_{ds}$ at different values of $\delta$**

| Data 1 | | Data 2 | | Data 3 | |
|---|---|---|---|---|---|
| $\delta$ | ***PRE*** | $\delta$ | ***PRE*** | $\delta$ | ***PRE*** |
| 0.33333 | 100 | 0.333333 | 100 | –0.56 | 100.13 |
| 0.4 | 101.81 | 0.4 | 109.77 | –0.5 | 110.33 |
| 0.5 | 104.04 | 0.444 | 111.87 | –0.25 | 150.42 |
| 0.6 | 105.59 | 0.55604 | 100 | –0.11372 | 158.62 |
| 0.7 | 106.4 | 0.60 | 90.88 | 0 | 152.82 |
| 0.75 | 106.52 | - | - | 0.25 | 114.28 |
| 0.8 | 106.44 | - | - | 0.3 | 105.6 |
| 0.9 | 105.7 | - | - | 0.33333 | 100 |
| 1 | 104.22 | - | - | 0.40 | 89.41 |
| 1.176595 | 100 | - | - | - | - |

**Table 3: *PRE* of $\hat{\hat{\bar{Y}}}_{(1)}$ with respect to $\hat{\hat{\bar{Y}}}_{R(ds)}$ at different values of $\delta$**

| Data 1 | | Data 2 | | Data 3 | |
|---|---|---|---|---|---|
| $\delta$ | ***PRE*** | $\delta$ | ***PRE*** | $\delta$ | ***PRE*** |
| –0.3333 | 100 | –0.33333 | 100 | –0.33333 | 100 |
| -0.25 | 104.67 | -0.25 | 120.91 | -0.3 | 103.6 |
| 0 | 118.63 | 0 | 234.87 | –0.25 | 108.25 |
| 0.25 | 131.16 | 0.25 | 498.58 | –0.2 | 111.71 |
| 0.5 | 140.09 | 0.5 | 660.16 | –0.1 | 114.08 |
| 0.75 | 143.42 | 0.75 | 359.07 | 0 | 109.97 |
| 1 | 140.34 | 1 | 171.93 | 0.10589 | 100 |
| 1.25 | 131.6 | 1.22271 | 100 | 0.15 | 94.81 |
| 1.5 | 119.17 | 1.25 | 94.26 | - | - |
| 1.75 | 105.23 | - | - | - | - |
| 1.8432 | 100 | - | - | - | - |

**Table 4: *PRE* of $\hat{\hat{\bar{Y}}}_{(1)}$ with respect to $\hat{\hat{\bar{Y}}}_{P(ds)}$ at different values of $\delta$**

| Data 1 | | Data 2 | | Data 3 | |
|---|---|---|---|---|---|
| $\delta$ | ***PRE*** | $\delta$ | ***PRE*** | $\delta$ | ***PRE*** |
| 0.50993 | 100 | –0.11062 | 100 | –1.22744 | 100 |
| 0.6 | 101.31 | 0 | 136.61 | –1 | 140.38 |
| 0.7 | 102.09 | 0.25 | 289.99 | –0.75 | 212.03 |
| 0.8 | 102.13 | 0.5 | 383.97 | –0.5 | 322.61 |
| 0.9 | 101.42 | 0.75 | 208.85 | –0.25 | 439.84 |
| 1 | 100 | 1 | 100 | 0 | 446.85 |
| 1.25 | 93.77 | 1.25 | 54.83 | 0.25 | 334.14 |
| - | - | - | - | 0.5 | 220.36 |
| - | - | - | - | 0.75 | 145.48 |
| - | - | - | - | 1 | 100 |

**Table 5: *PRE* of $\hat{\hat{\bar{Y}}}_{(1)}$ with respect to $\hat{\hat{\bar{Y}}}_{\mathrm{Re}(ds)}$ at different values of $\delta$**

| Data 1 | | Data 2 | | Data 3 | |
|---|---|---|---|---|---|
| $\delta$ | ***PRE*** | $\delta$ | ***PRE*** | $\delta$ | ***PRE*** |
| 0 | 100 | 0 | 100 | –0.2274 | 100 |
| 0.25 | 110.56 | 0.1 | 135.36 | –0.2 | 101.58 |
| 0.5 | 118.08 | 0.2 | 183.9 | –0.1 | 103.74 |
| 0.75 | 120.9 | 0.25 | 212.28 | 0 | 100 |
| 1 | 118.29 | 0.3 | 241.01 | 0.1 | 91.53 |
| 1.25 | 110.93 | 0.5 | 281.08 | - | - |
| 1.5099 | 100 | 0.6 | 235.03 | - | - |
| 1.65 | 93.45 | 0.7 | 178.14 | - | - |
| - | - | 0.88937 | 100 | - | - |

**Table 6: *PRE* of $\hat{\bar{Y}}_{(1)}$ with respect to $\hat{\bar{Y}}_{Pe(ds)}$ at different values of $\delta$**

| Data 1 | | Data 2 | | Data 3 | |
|---|---|---|---|---|---|
| $\delta$ | ***PRE*** | $\delta$ | ***PRE*** | $\delta$ | ***PRE*** |
| 0.66667 | 100 | 0.22271 | 100 | –0.89411 | 100 |
| 0.7 | 100.17 | 0.25 | 107.99 | –0.75 | 127.37 |
| 0.75 | 100.28 | 0.3 | 122.6 | –0.5 | 193.81 |
| 0.8 | 100.21 | 0.4 | 144.41 | –0.25 | 264.23 |
| 0.84326 | 100 | 0.5 | 142.98 | 0 | 268.44 |
| 0.90 | 99.52 | 0.6 | 119.56 | 0.25 | 200.73 |
| - | - | 0.66666 | 100 | 0.5 | 132.38 |
| - | - | 0.70 | 90.62 | 0.66666 | 100 |

The optimum value of $\delta$ and the range of $\delta$ in which the proposed estimator $\hat{\bar{Y}}_{(1)}$ dominates over the estimators $\bar{y}_{ds}$, $\hat{\bar{Y}}_{R(ds)}$, $\hat{\bar{Y}}_{P(ds)}$, $\hat{\bar{Y}}_{\mathrm{Re}(ds)}$ and $\hat{\bar{Y}}_{Pe(ds)}$ are displayed in Table 1.

The percent relative efficiencies (*PRE*s) of the suggested estimator $\hat{\bar{Y}}_{(1)}$ with respect to $\bar{y}_{ds}$, $\hat{\bar{Y}}_{R(ds)}$, $\hat{\bar{Y}}_{P(ds)}$, $\hat{\bar{Y}}_{\mathrm{Re}(ds)}$ and $\hat{\bar{Y}}_{Pe(ds)}$ for varying values of $\delta$ are presented in Tables 2 to 6, respectively.

It is observed from Tables 2 to 6 that the proposed estimator $\hat{\bar{Y}}_{(1)}$ is more efficient than the estimators $\bar{y}_{ds}$, $\hat{\bar{Y}}_{R(ds)}$, $\hat{\bar{Y}}_{P(ds)}$, $\hat{\bar{Y}}_{\mathrm{Re}(ds)}$ and $\hat{\bar{Y}}_{Pe(ds)}$ in certain range of $\delta$ as given in Table 1. Further, we observed that there is largest gain in efficiency by using the estimator $\hat{\bar{Y}}_{(1)}$ over the estimators $\bar{y}_{ds}$, $\hat{\bar{Y}}_{R(ds)}$, $\hat{\bar{Y}}_{P(ds)}$, $\hat{\bar{Y}}_{\mathrm{Re}(ds)}$ and $\hat{\bar{Y}}_{Pe(ds)}$ for all the data sets 1, 2 and 3 at the optimum value of $\delta$.

Table 2 exhibits that the proposed estimator $\hat{\bar{Y}}_{(1)}$ is more efficient than the usual unbiased estimator $\bar{y}_{ds}$ for a wider range of $\delta$ for data sets 1 and 3 while it is better than $\bar{y}_{ds}$ for a shorter range of $\delta$. For data set 3, the gain in efficiency is also substantial by using the estimator $\hat{\bar{Y}}_{(1)}$ over $\bar{y}_{ds}$.

It is observed from Table 3 that the estimator $\hat{\bar{Y}}_{(1)}$ gives the largest amount of gain in efficiency over ratio estimator $\hat{\bar{Y}}_{R(ds)}$ for a broad range of $\delta$ for the data sets 1 and 2. While it is marginal for a smaller range of $\delta$ for the data set 3.

Table 4 presents that the estimator $\hat{\bar{Y}}_{(1)}$ is more efficient than the product estimator $\hat{\bar{Y}}_{P(ds)}$ for a broader range of $\delta$ with substantial gain in efficiency for data set 3. It also presents considerable gain in efficiency by using $\hat{\bar{Y}}_{(1)}$ over the product estimator $\hat{\bar{Y}}_{P(ds)}$ but for a smaller range of $\delta$ for data set 2. The gain in efficiency is marginal for data set 1.

Table 5 demonstrate that the suggested estimator $\hat{\bar{Y}}_{(1)}$ is more efficient than ratio-type exponential estimator $\hat{\bar{Y}}_{\mathrm{Re}(ds)}$ with substantial gain in efficiency for a wide range of $\delta$ for data sets 1 and 2, while it is marginal for data set 3.

It is observed from Table 6 that there is sizable gain in efficiency by using the estimator $\hat{\bar{Y}}_{(1)}$ over the product-type exponential estimator $\hat{\bar{Y}}_{Pe(ds)}$ for a wide range of $\delta$ for data sets 2 and 3 while it is marginal in case of population data set 1.

Finally, we conclude from Tables 1 to 6 that even if the scalar $\delta$ deviates from its "exact optimum value" the gain in efficiency by using $\hat{\bar{Y}}_{(1)}$ over $\bar{y}_{ds}$, $\hat{\bar{Y}}_{R(ds)}$, $\hat{\bar{Y}}_{P(ds)}$, $\hat{\bar{Y}}_{\mathrm{Re}(ds)}$ and $\hat{\bar{Y}}_{Pe(ds)}$ is considerable. Also, there is enough scope of selecting the values of $\delta$ for obtaining better estimators than existing estimators. Thus, we recommend the use of the proposed estimator $\hat{\bar{Y}}_{(1)}$ in practice.

## 6. Discussion

In this article, we have discussed the problem of estimating the population mean using auxiliary information in double sampling for stratification. A class of product-cum-ratio-type estimators $\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)}$ have been developed. Expressions of bias and mean squared error of the developed estimator $\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)}$ have been derived up to the first order of approximation. Optimum condition is derived at which the mean squared error of the proposed estimator $\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)}$ is minimized. In particular, to demonstrate the utility of the general results, we consider a subclass of the developed estimator $\hat{\bar{Y}}^{(c)}_{P\mathrm{Re}(ds)}$ named as 'product-cum-ratio-type exponential' estimator $\hat{\bar{Y}}_{(1)}$. Properties of the subclass of estimators $\hat{\bar{Y}}_{(1)}$ have been studied. Regions of preferences have been derived in which the suggested subclass of estimators $\hat{\bar{Y}}_{(1)}$ is more efficient than the usual unbiased estimator $\bar{y}_{ds}$, Ige and Tripathi (1987) ratio estimator $\hat{\bar{Y}}_{R(ds)}$ and product estimator $\hat{\bar{Y}}_{P(ds)}$, Tailor *et al.* (2014) ratio-type exponential estimator $\hat{\bar{Y}}_{\mathrm{Re}(ds)}$ and product-type exponential estimator $\hat{\bar{Y}}_{Pe(ds)}$. We have also carried out an empirical study to demonstrate the performance of the proposed estimator $\hat{\bar{Y}}_{(1)}$ over other existing estimators.

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