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Accelerated Life Test Acceptance Sampling Plans for the Weibull Distribution with Constant Acceleration using EWMA and Modified EWMA Statistics

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Abstract

Products or systems with high reliability and durability require more time for inspection as it is difficult to detect failures under normal operating conditions. In literature, accelerated life testing methods are discussed to accelerate the decision, in which products are tested under severe than normal stress conditions with failure mechanisms similar to the one observed in the field. The most commonly applied stressis constant stress. We propose life test sampling plans for establishing a quantile life for the Weibull distribution under a constant acceleration factor. As the accelerated life test plans require more sample size than those without acceleration, the previous history has been used as per EWMA and Modified EWMA schemes to make them somewhat economical. Tables of optimal design parameters are presented for three different acceleration constants for establishing a Weibull median life. Published real-life data sets are used to demonstrate the proposed life test sampling plans.

Key words: Acceleration Factor; Quantile; Weibull Distribution; EWMA; Modified EWMA.

AMS Subject Classifications: 62P30

1. Introduction

The era of a competitive market with rapid developments in technology motivates manufacturers to supply products with high durability and reliability. To achieve this dimension of quality, in literature, online, and offline product control techniques are discussed in which the acceptance sampling or product control is an offline technique for deciding to accept or reject a lot through sample inspection before sending them to market.The most popular acceptance sampling plan is the single sampling plan in which a random sample of size (n) is taken from a lot of size (N) and number of defectives (d) are observed, and the decision to accept the lot if $d \leq c$ (acceptance number) otherwise reject the lot. The concept and usage of acceptance sampling were first narrated by Dodge and Romig (1941) for inspecting bullets.

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The acceptance sampling plans are developed based on life tests known as life test sampling plans (LSP). To reduce the cost of testing, a sample of products is put on life test for a time smaller than the specified mean or quantile life, when the mean or quantile life of products exceeds the specified mean or quantile life, the lot of products under inspection is accepted otherwise it is rejected. Several authors have given LSPs which are based on mean lifetimes, to mention are Epstein (1954) gave those for the exponential distribution, Goode and Kao (1960 and 1961) for Weibull distribution, Baklizi and Masri (2004) for Birnbaum Model, Rosaiah and Kantam (2005) for inverse Rayleigh distribution, Tsai and Wu (2006) for Generalized Rayleigh distribution, and, Khan and Alqarni (2020) for inverse Weibull distribution.

Engineers focus more on the product's percentile life during life testing applications than its mean life. The reason is that the mean may not be an appropriate measurement when one perceives a product's quality to be in the low percentile. A small decrease in mean with a simultaneous small increase in variance can result in a downward shift in small percentiles of interest. Gupta (1962), Balakrishnan *et al.* (2007), Singh *et al.* (2015)and Singh and Tripathi (2017) gave time-truncated attribute LSPs for Normal and Log-Normal, generalized Birnbaum-Saunders distribution, generalized inverted exponential distribution, inverse Weibull distribution respectively for median lifetimes. Lio *et al.* (2009), Rao and Kantam (2010), Aslam *et al.* (2011), Rao *et al.* (2012), Rao *et al.* (2013), Rao and Rao (2013), Rao (2013), Malathi and Muthulakshmi (2016), Kaviyarasu and Fawaz (2017), Pradeepaveerakumari and Ponneeswari (2017) and Raykundaliya *et al.* (2022) developed LSPs assuring percentile lifetime of a product when life time distribution are respectively Birnbaum-Saunders, Log-logistic Burr type XII, inverse Rayleigh distribution, Half Normal, generalized Logistic, Marshall-Olkin extended exponential, Gompertz, Modified Weibull, Exponential Rayleigh, and Weibull.

In a competitive environment, to maintain a brand name, one needs to produce products of high reliability and durability. Such products not only require more time for inspection but also fail to detect failures under normal operating conditions. Industries of the above nature require a mechanism that reduces the time required for testing. Accelerated testing reduces testing time and helps to draw quick inferences about the product being tested, that is products under inspection are subjected to higher than usual stress. Nelson (1990) gave the basic idea of accelerated life testing and graphical analysis to estimate product life. Lin and Chiu (1995) constructed a cost model for an accelerated life test sampling plan. Escobar and Meeker (2006) have outlined some of the basic ideas behind accelerated testing. Xiaoyang *et al.* (2015) developed accelerated life testing plans considering lognormal distribution. Aslam et al (2019) gave accelerated LSPs by studying the mean life of Weibull distribution.

The scale family of distributions plays a very important role in lifetime data analysis. Among them the Weibull distribution is widely used in reliability studies because of its flexibility to take various shapes and, IFR - DFR properties. It is popular in warranty analysis, utility services, and industries manufacturing bearings, capacitors, because lifetimes under accelerated testing continue to follow the Weibull distribution with nearly the same shape parameter. This makes development of accelerated LSPs for assuring a specified life of Weibull distribution mathematically tractable. However, the accelerated LSPs require a higher sample size than the usual LSPs because of collecting only restricted lifetime data.

The sample size can be reduced by considering weighted average of the current lot information with the past lot information (see Aslam *et al.*, 2017). Using this approach, Divecha and Raykundaliya (2020) have designed LSPs as per EWMA and Modified EWMA, and showed that they are more economical than those developed based on only current lot information. In this paper, we propose LSPs for assuring quantile Weibull lifetime under constant acceleration using EWMA and Modified EWMA for early decision with lesser sample size. The organization of the paper is as follows:

In Section 2, we briefly discuss the Weibull distribution in the context of the newer term, constant acceleration factor (AF). In Section 3, we discuss the proposed LSPs for the Weibull distribution with acceleration (WDwALSP) followed by economical WDwALSP based on the EWMA and Modified EWMA.In Section 4, we illustrate the LSPs and give hypothetical example. In Section 5, we give two real-life examples to demonstrate the use of proposed plans and compared them with LSPs without acceleration. Concluding remarks are given in Section 6.

2. Weibull distribution and probability of failure under accelerated quantile life ratio

The cumulative distribution function of the Weibull distribution is defined by

$$F(t) = 1 - e^{-(\sigma t)^{\theta}};\ t > 0,\ \sigma > 0,\ \theta > 0, \tag{1}$$

where θ and σ are respectively shape and scale parameters of the distribution.

Let t_U denotes lifetime of a product under normal conditions following Weibull distribution with CDF,

$$F(t_U) = 1 - e^{-(\sigma_U t_U)^{\theta}}. \tag{2}$$

It's 100q$^{\text{th}}$ quantile life time is $t_{qU} = \frac{(-\log(1-q))^{\frac{1}{\theta}}}{\sigma_U} = \frac{b}{\sigma_U}$, where $b = (-log(1-q))^{\frac{1}{\theta}}$.

Therefore,

$$\sigma_U = \frac{b}{t_{qU}}. \tag{3}$$

Let t_A be the lifetime of a product under constant stress (acceleration) having Weibull distribution with CDF,

$$F(t_A) = 1 - e^{-(\sigma_A t_A)^{\theta}} \text{ with } \sigma_A = \frac{b}{t_{qA}}. \tag{4}$$

It's 100q$^{\text{th}}$ quantile life is $t_{qA} = \frac{b(AF)}{\sigma_A}$, where, $AF = \frac{t_U}{t_A}$,
implies

$$t_A = \frac{t_U}{AF} \tag{5}$$

and

$$\frac{\sigma_A}{\sigma_U} = \frac{(AF)t_{qU}}{t_{qA}} \tag{6}$$

Let

$$\frac{\sigma_A}{\sigma_U} = RAR \text{ (ratio of acceleration rate, } RAR > AF) \tag{7}$$

Let, $\tau_A = \frac{t_{U0}}{AF}$, where t_{U0} is truncation time under normal conditions defined as $t_{U0} = \delta_0 t^0_{qU}$, $0 < \delta_0 < 1$ is a constant called termination ratio, and $\frac{t_{qA}}{t^0_{qA}}$, is the ratio of true quantile lifetime to the specified quantile lifetime representingthe quality of a lot. Using (6), $\tau_A = \frac{\delta_0\left(\frac{\sigma_A}{\sigma_U}\frac{1}{AF}t^0_{qA}\right)}{AF}$ for which the CDFin terms of AF and RAR, using (4) and (7) is,

$$F(\tau_A) = 1 - e^{-\left((b)^\theta(\delta_0)^\theta(RAR)^\theta\left(\frac{1}{AF}\right)^\theta\left(\frac{t_{qA}}{t^0_{qA}}\right)^{-\theta}\right)} \tag{8}$$

The specified accelerated quantile life t^0_{qA}, lot quality ratio $\left(\frac{t_{qA}}{t^0_{qA}}\right)$ (> 1), accelerated test termination time truncation ratio δ_0 (< 1), AF (> 1), and RAR $(>$ AF$)$ are decided by the producer.

Further, we know that H_0: $t_{qU} \geq t^0_{qU}$ Vs H_1: $t_{qU} < t^0_{qU}$ and H_0: $t_{qA} \geq t^0_{qA}$ Vs H_1: $t_{qA} < t^0_{qA}$ are equivalent and let truncated time for acceleratedlife tests as τ_A.

3. Procedure of LSPs for the Weibull distribution with acceleration (WD-wALSP)

Suppose that producer submit a lot of units for accelerated testing, whose lifetimes follow the Weibull (θ, σ) and claims that the true quantile life t_{qA} of the lot is better than the specified qaunitle life t^0_{qA}. To support producer's claim, we propose the design of a time truncated LSPs based on accelerated quantile lifetime, with following procedure:

Step1: Suppose 'n' items from a lot are taken and put on life test until accelerated test time τ_A and observe lifetimes X_j $(j = 1, 2, \ldots, n)$.

Step 2: Define an indicator variable say, I_j;$(j = 1, 2, \ldots, n)$

$$I_j = \begin{cases} 1, & \text{if } 0 < X_j \leq \tau_A,\ j = 1, 2, \ldots, n \\ 0, & \text{Otherwise} \end{cases} \tag{9}$$

Let 'd' denote failed items during test time $(0, \tau_A]$. Define $d = \sum_{j=1}^{n} I_j$.

Step 3: If $d \leq c$, accept the lot, otherwise reject the lot.

While inspecting items with acceleration during the time interval $(0, \tau_A]$, the sample under study may fail with probability p where $p = F(\tau_A)$, given by (8), and the count of failed items'd' follows the binomial distribution having parameters n and p, with mean np and variance $np(1-p)$. A lot must be accepted if the data support the null hypothesis H_0 : $t_{qA} \geq t^0_{qA}$ against the alternative hypothesis H_1 : $t_{qA} < t^0_{qA}$ satisfying both producer's and consumer's requirements.

The design parameters (n, c) are to be obtained by solving the following two inequalities simultaneously.

$$\begin{aligned} L(p_1) &= \sum_{d=0}^{c} \binom{n}{d} p_1^d (1-p_1)^{n-d} \geq 1 - \alpha \\ L(p_2) &= \sum_{d=0}^{c} \binom{n}{d} p_2^d (1-p_2)^{n-d} \leq \beta \end{aligned} \tag{10}$$

where, $p_1 = F(\tau_A)$ at $\frac{t_{qA}}{t^0_{qA}} > 1$ and $p_2 = F(\tau_A)$ at $\frac{t_{qA}}{t^0_{qA}} = 1$ denotes the probability of failure for a good lot and for a poor lot during the time $(0, \tau_A]$.

Under normal approximation, binomial probabilities given in equation (10) representing producer's and consumer's requirements are given in terms of standard normal distributions Φ as

$$\begin{aligned} \Phi\left(\frac{c-np_1}{\sqrt{np_1(1-p_1)}}\right) \geq 1-\alpha \\ \Phi\left(\frac{c-np_2}{\sqrt{np_2(1-p_2)}}\right) \leq \beta \end{aligned} \tag{11}$$

The normal approximation to the binomial is known to be satisfactory for p_a approximately $\frac{1}{2}$ and $n > 10$. In general, if $\frac{1}{n+1} < p_A < \frac{n}{n+1}$ then the normal approximation is adequate. In numerical form, if $np_A > 10$ and $0.1 \leq p_A \leq 0.9$ then normal approximation is appropriate to the binomial distribution. Referto Montgomery (2001) (Page no. 76-77).

3.1. WDwALSP using EWMA

Accelerated LSP almost requires more than double the sample size, thereby increasing the cost of inspection, than those by the standard LSP that is without accelerated tests. Therefore, there has to be a procedure by which it can be brought down to the sample size of the standard LSPs. Use of EWMA statistics to summarize the previous lot inspection number of failures with the current one brings down the sample size effectively for Weibull (Aslam, 2017), and Generalized Exponential distributions (Divecha and Raykundaliya, 2018). Therefore, if an industry maintains a database of inspection history of lots, step3 in section 3 would be as follows.

Step 3: Calculate the EWMA statistic of Roberts (1959), based on d_l for $l = 1, 2, \ldots$

$$EWMA_l = \lambda d_l + (1-\lambda) EWMA_{l-1};\ EWMA_1 = d_1 \tag{12}$$

where $0 < \lambda < 1$, λ is a smoothing constant chosen because. if $\lambda = 0.6$ means the weight attached to the current sample is 0.6 and the weights attached to the past sample is 0.4, the current number of failures will be affected not by more than the previous 5 lot information.

Then, Accept the lot if $EWMA_l \leq c$, otherwise, reject the lot.

Following Montgomery (2001) the mean and variance of $EWMA$ statistic are given by,

$$\mu_{EWMA} = np$$

and

$$\sigma^2_{EWMA} = np(1-p)\left(\frac{\lambda}{2-\lambda}\right) \tag{13}$$

Therefore, the WDwALSP-based EWMA design consists of estimating (n, c), subject to the producer's and consumer's requirements in terms of probabilities of accepting the lot as those given in (10) are

$$\begin{aligned} L(p_1) = P(EWMA_l \leq c) \\ L(p_2) = P(EWMA_l \leq c) \end{aligned} \tag{14}$$

After normalization and using (13), (14) becomes,

$$L(p) = \Phi\left(\frac{c - \mu_{EWMA}}{\sigma_{EWMA}}\right) \tag{15}$$

with $p = p_1$ and p_2 appropriately.

The probability of accepting a good and poor lot using (15) are

$$\begin{aligned} L(p_1) &= \Phi\left(\frac{c-np_1}{\sqrt{np_1(1-p_1)\left(\frac{\lambda}{2-\lambda}\right)}}\right) \geq 1-\alpha \\ L(p_2) &= \Phi\left(\frac{c-np_2}{\sqrt{np_2(1-p_2)\left(\frac{\lambda}{2-\lambda}\right)}}\right) \leq 1-\beta \end{aligned} \tag{16}$$

This pair of equations is used to determine the WDwALSP parameters under EWMA.

3.2. WDwALSP using modified EWMA

A further reduction in sample size for WDwALSP is possible by using Modified EWMA (Khan *et al.*, 2016) as shown in Divecha and Raykundaliya (2020) as the Modified EWMA statistic has a variance smaller than that of EWMA.

Step 3 in Section 3 is taken as follows.

Step 3: Calculate Modified EWMA statistic

$$MEWMA_l = \lambda/2(d_l + d_{l-1}) + (1-\lambda)MEWMA_{l-1};\ MEWMA_1 = d_1 \tag{17}$$

where, $0 < \lambda < 1$; $l > 1, \lambda$ is a smoothing constant chosen just as in the EWMA scheme.

If $MEWMA_l \leq c$, Accept the lot, otherwise, reject the lot.

The mean and variance of Modified EWMA statistic are $\mu_{MEWMA} = np$

and

$$\sigma^2_{MEWMA} = np(1-p)\left(\frac{\lambda}{2}\right) \tag{18}$$

Similar to Section 3.1, the probabilities of accepting a good and poor lot are

$$\begin{aligned} L(p_1) &= \Phi\left(\frac{c-np_1}{\sqrt{np_1(1-p_1)\left(\frac{\lambda}{2}\right)}}\right) \geq 1-\alpha \\ L(p_2) &= \Phi\left(\frac{c-np_2}{\sqrt{np_2(1-p_2)\left(\frac{\lambda}{2}\right)}}\right) \leq \beta \end{aligned} \tag{19}$$

This pair of equations is used to determine the WDwALSP parameters under modified EWMA.

4. Constructions of tables of proposed LSP

Tables 2 – 4 of WDwALSP are constructed by satisfying (11) to establish the 50^{th} quantile life with fixed Producer's risk $\alpha = 0.05$, consumer's risks $(\beta = 0.25, 0.10, 0.05, 0.01)$, shape parameters $(\theta = 1.5, 2.0, 2.5)$, quality levels $\left(\frac{t_{qA}}{t^0_{qA}} = 1.25, 1.50, 1.75, 2.00, 2.25\right)$, termination ratios $(\delta_0 = 0.6, 0.8)$, acceleration factors (AF=1.5,2.0,2.5) and the ratio of acceleration rate(RAR=2.0,2.5,3.0).

Tables 5 – 13 of WDwALSP using EWMA and Tables 14 – 22 of WDwALSP using Modified EWMA are obtained by satisfying equations (16) and (19) with the above parameters and smoothing constant $(\lambda = 0.2, 0.4, 0.6, 0.8, 1.0)$ as per EWMA and Modified EWMA statistic. All tables are constructed using R 3.6 language.

From all of the constructed tables, the following observations are made:

1. Keeping fixed producer risk, with the increase in termination ratio (that is increase the truncation time), quality level (that is true process quantile lifetime is much higher than specified quantile lifetime), consumer risk, shape parameter (that is probability of failure of a product is increased) as a result there is a decrease in sample size and acceptance number.

2. With the increase in the smoothing constant, there is an increase in sample size and acceptance number. It means, if information about the quality history of a lot is less to the producer, for testing of a lot, higher sample size is required, and hence cost of inspection of a lot is increased.

Among the comparison of all constructed tables of WDwALSP, WDwALSP with EWMA, and WDwALSPwithModified EWMA lesser sample sizes are required under the last type.

Usually,the question arises in our mind what should be sample size required for inspection when acceleration is given and not given? Which of the LSPs is better? To address these questions, we give tables for optimal design parameters for LSP for Weibull distribution (WDLSP) and LSP for Weibull distribution with acceleration (WDwALSP) with EWMA and modified EWMA respectively in tables 23 - 25 and 2 - 4 for different shape parameters and different process parameters. The Tables 23 - 25 obtains using (11), (16) and (19) after substituting AF=1 and RAR=1 in (8). From both plans' tables, we observed that somewhat higher sample size is required for the inspection of a lot in WDwALSP compared to WDLSP.Lin and Chiu (1995) showed in their paper using an example that, accelerated plans required a higher sample size in comparison to the plans without acceleration. They also showed that the overall cost of the experiment and time of experiments are reduced compared to plans without acceleration. From Tables (2-4), it is observed that the WDwALSP with EWMA or modified EWMA requires a moderately lesser sample size than the WDLSP. Hence under acceleration, plans with EWMA or modified EWMA are economical and preferable provided inspection histories of lots are available.

5. Illustration

We use two data sets, Lawless (1982, Page 228) ball bearing data and Murthy *et al.* (2004) repairable item data to illustrate WDwALSP introduced in this paper.

5.1. Lawless dataset

A million revolutions before failure for each of the 23 ball bearings tested were as follows.

17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40.

The maximum likelihood estimators of shape and scale parameters of the well-fitted Weibull distribution ($AIC = 231.37$, KS-test $D = 0.1502(p = 0.62)$) for the number of million revolutions before failure were respectively,2.1014 (considered as 2) and 0.0122.

5.1.1. WDLSP

We illustrate the proposed LSP with and without acceleration using the ball bearing data. We estimate the 50th quantile of lifetimes, which is 67.80. Suppose producer and consumer with specified risks (0.05, 0.1) agrees to accept the lot if the true median life is 1.75 times better than the specified median life, which would be 38.75. Further, they agree on test termination time to be 0.8 of the specified median time, which results in 31. The LSP design parameters for this set of constants are read from the Table 23. So, the parameters (n, c) of chosen LSP is (12,2). During this test time, the total number of failed items in the given sample is 2, which is equal to the acceptance number hence, the lot is accepted.

5.1.2. WDLSP with EWMA and modified EWMA

When the previous history of lot inspection is available, the user can take advantage and inspect the lot with lesser items on the test.

Since the history of data is not available, we have generated four samples (using R Language) of size 6 having shape parameter $\hat{\theta} = 2.1014$ and $\hat{\sigma} = 0.0122$

Under the same set of constants as above the parameter of WDLSP with EWMA from Table 24 are $(n = 6, c = 1)$, requiring only 6 items to be tested instead of 10 items. To illustrate this, we suppose the lifetimes of previous lot inspections are not available, hence we simulate four samples of Weibull distributed lifetimes having shape parameter 2 and scale parameter 0.01 each of size 6, which are:

41.69, 59.15, 39.53, **15.26**, 52.65, **23.28**
124.89, 42.78, 44.61, **24.58**, 139.28, 55.40
139.65, **17.83**, 87.01, 88.02, **23.83**, **26.95**
78.04, 32.49, 81.28, **11.78**, 51.21, 58.62

Real-life data: 17.88, 28.92, 33.00, 41.52, 42.12, 45.60

Their inspection results in the number of failures as $d_1 = 2$, $d_2 = 1$, $d_3 = 3$, $d_4 = 1$,

$d_5 = 2$. As per (12), the corresponding EWMA statistics values are $\boldsymbol{EWMA_1 = 2}$, $\boldsymbol{EWMA_2 = 1.40}$, $\boldsymbol{EWMA_3 = 2.36}$, $\boldsymbol{EWMA_4 = 1.54}$, and $\boldsymbol{EWMA_5 = 1.81}$, respectively. Since $\boldsymbol{EWMA_5 > 1}$, the decision would bea current lot is rejected. Notice that, a bad history may change the decision.

Modified EWMA statistic uses the historical information slightly differently and may further reduce the sample size needed to establish the quality and decision about a lot. Observe that under the same set of constants as above the parameter of WDLSP with Modified EWMA from Table 25 are $(n = 5, c = 1)$, less by one more item than needed by WDLSP under EWMA. The number of failures as $d_1 = 1$, $d_2 = 1$, $d_3 = 2$, $d_4 = 1$, $d_5 = 2$ As far as the decision process is concerned, we calculate as per (17) the Modified EWMA statistics and find them as $\boldsymbol{MEWMA_1 = 1}$, $\boldsymbol{MEWMA_2 = 1}$, $\boldsymbol{MEWMA_3 = 1.3}$, $\boldsymbol{MEWMA_4 = 1.42}$, and $\boldsymbol{MEWMA_5 = 1.468}$, respectively. Since $\boldsymbol{MEWMA_5 = 1.468 > c = 1}$, The decision is once again to reject the lot because of a bad third lot.

5.1.3. WDwALSP

We extend the use of ball bearing data to illustrate the LSPs for Weibull distribution under acceleration factor $AF = 2$. The data set is transformed accordingly (dividing by 2), so they are,

8.94, 14.46, 16.5, 20.76, 21.06, 22.8. 24.4, 25.92, 25.98, 27.06, 27.78, 33.9, 34.22, 34.32, 34.44, 42.06, 46.56, 49.32, 52.56, 52.92, 63.96, 64.02, 86.7.

The accelerated 50$^{\text{th}}$ quantile of failure time is $t_{0.5A} = 33.9$. We need to specify the ratio $RAR = \frac{\sigma_A}{\sigma_U} > 2$. ($\because$ Eq. 6 and Eq. 7). The test time to establish a true median life of 1.75 better than the specified, under termination ratio ratio $\delta_0 = 0.8$, and $RAR = 2.5$ would be $\tau_A = 15.49$ units. The WDwLSP(n, c) for consumer risk $\beta = 0.1$, and other constants $(\hat{\theta}, \frac{t_{qA}}{t^0_{qA}}, \delta_0, AF, RAR) = (2, 1.75, 0.8, 2, 2.5)$, are (23,8) as per Table 3. The sample inspection shows that two items have failed. Since $d = 2 < c = 8$, the lot is accepted.

Use of previous lot inspection data and use of EWMA or Modified EWMA statistic to judge a lot is advisable because the sample size increases marginally with an increase in the acceleration factor (See Tables 1-3).

5.1.4. WDwALSP with EWMA and modified EWMA

For consumer risk $\beta = 0.1$, the value of (n, c) for $(\hat{\theta}, \frac{t_{qA}}{t^0_{qA}}, \delta_0, AF, RAR, \lambda) = (2, 1.75, 0.8, 2, 2.5, 0.6)$ from the Table 9 is $(n = 11, c = 4)$

Since the history of data is not available, we have generated four Weibull lifetimes samples of size 11 having shape parameter $\hat{\theta} = 2.1014$ and $\hat{\sigma} = 0.0122$ in R. They are,

20.84, 29.57, 19.76, **7.63**, 26.32, **11.64**, 45.57, 42.43, 66.55, 27.68, 38.695
62.44, 21.39, 22.30, **12.29**, 69.64, 27.7, 16.46, 24.62, 18.40, 39.91, 50.43
69.82, **8.91**, 43.50, 44.01, **11.91**, **13.47**, 38.69, 39.59, 33.165, 42.65, 35.24
39.02, 16.24, 40.64, **5.89**, 25.60, 29.31, 18.97, 43.48, 20.44, 16.01, 23.21

Real-life data: 8.94, 14.46, 16.5, 20.76, 21.06, 22.8. 24.4, 25.92, 25.98, 27.06, 27.78

The inspection of the past 4 lots results in the following number of failures from 5 items using (2) **we have:** $\boldsymbol{d_1 = 2,\ d_2 = 1,\ d_3 = 3,\ d_4 = 1}$ while the current lot is having $\boldsymbol{d_5 = 2}$. As per (12), the corresponding EWMA statistics values are $\boldsymbol{EWMA_1 = 2,\ EWMA_2 = 1.40,\ EWMA_3 = 2.36,\ EWMA_4 = 1.54}$, and $\boldsymbol{EWMA_5 = 1.81}$, respectively. Since $\boldsymbol{EWMA_5 = 1.8176 < c = 4}$, the decision would be a current lot is accepted.

For the same set of constraints, WDwA with Modified EWMA from Table 18 the value of (n, c) is (8,3). Also, the inspection of the past 4 lots results in the following number of failures from 8 items: $\boldsymbol{d_1 = 2,\ d_2 = 1,\ d_3 = 3,\ d_4 = 1}$ while the current lot i.e., the 5th lot is having some failures items as $\boldsymbol{d_5 = 2}$. We calculate as per (17) the Modified EWMA statistics and find them as $\boldsymbol{MEWMA_1 = 2,\ MEWMA_2 = 1.7,\ MEWMA_3 = 1.88,\ MEWMA_4 = 1.952}$, and $\boldsymbol{MEWMA_5 = 1.6808}$ respectively. Since $\boldsymbol{MEWMA_5 = 1.6808 < c = 3}$, the decision is once again accepted. 11.5

Table 1: Summary: Comparison of ball bearing data by considering with and without acceleration

	Acceleration Factor	**50th Quantile**	**Specified Quantile**	**Termination Time**	**Table Values**	**Conclusion**
WDwLSP	$AF = 2$	$t_{qA} = 33.9$	$t_{qA}^0 = 19.37$	$\tau_A = 15.496$	(23,8)	Accept the Lot
WDwALSP with EMMA					(11,4)	
WDwALSP with Modified EMMA					(8,3)	
WDLSP	$AF = 1$	$t_{qA} = 67.80$	$t_q^0 = 38.7428$	$t_0 = 30.9942$	(12,2)	Accept the Lot
WDLSP with EMMA					(6,1)	Reject the Lot
WDLSP with Modified EMMA					(5,1)	Reject the Lot

Murthy et. al. (2004) represent data from 30 observations about the time between failures for the repairable item. **1.43, 0.11, 0.71, 0.77, 2.63, 1.49, 3.46, 2.46, 0.59, 0.74, 1.23, 0.94, 4.36, 0.40, 1.74, 4.73, 2.23,0.45, 0.70, 1.06, 1.46, 0.30, 1.82, 2.37, 0.63, 1.23, 1.24, 1.97, 1.86, 1.17**

Arranging the data in ascending order

0.11, 0.3, 0.4, 0.45, 0.59, 0.63, 0.7, 0.71, 0.74, 0.77, 0.94, 1.06, 1.17, 1.23, 1.23, 1.24, 1.43, 1.46, 1.49, 1.74, 1.82, 1.86, 1.97, 2.23, 2.37, 2.46, 2.63, 3.46, 4.36, 4.73

The maximum likelihood estimators of shape and scale parameters of the Weibull distribution for the number of million revolutions before failure are respectively $\hat{\theta} = 1.4633 \cong 1.5$ and $\hat{\sigma} = 0.5848$. Further, $AIC = 83.8207$, KS-test value $D = 0.074869$, and the corresponding p-value is 0.996 which is greater than 0.05, suggesting that the Weibull distribution fits well with the data.

Transformed Original Data for the proposed plan by taking $\boldsymbol{AF = 2.5 \left(\because AF = \frac{t_U}{t_A}\right)}$

0.57, 0.04, 0.28, 0.31, 1.05, 0.60, 1.38, 0.98, 0.24, 0.30, 0.49, 0.38, 1.74, 0.16, 0.70, 1.89, 0.89, 0.18, 0.28, 0.42, 0.58, 0.12, 0.73, 0.95, 0.25, 0.49, 0.50, 0.79, 0.74, 0.47

Arranging data in ascending order:

0.04, 0.12, 0.16, 0.18, 0.24, 0.25, 0.28, 0.28, 0.30, 0.31, 0.38, 0.42, 0.47, 0.49, 0.49, 0.50, 0.57, 0.58, 0.60, 0.70, 0.73, 0.74, 0.79, 0.89, 0.95, 0.98, 1.05, 1.38, 1.74, 1.89

The accelerated 50$^{\text{th}}$ quantile of failure time is $t_{0.5A} = 0.495$ with shape parameter $\hat{\theta} = 1.5$ and let us take the ratio $\frac{\sigma_A}{\sigma_U} = RAR > AF \Rightarrow 3.0 > 2.5$

To establish the ratio $\frac{t_{qA}}{t^0_{qA}} > 1$, we take $\frac{t_{qA}}{t^0_{qA}} = 1.75 \Rightarrow t^0_{qA} = 0.2828$, also we take termination ratio $\delta_0 = 0.8$

$\tau_A = \delta_0 t^0_{qA} = 0.8(0.2828) = 0.2262$

Therefore, the total number of failed items in the given lot is $d = 4$. For consumer risk $\beta = 0.25$, the value of (n, c) for $(\hat{\theta}, \frac{t_{qA}}{(t^0_{qA}}, \delta_0, AF, RAR) = (1.5, 1.75, 0.8, 2.5, 3)$, from Table 2 is (25,10).

Here, $d = 4 < c = 10$ $(d < c)$, Hence, the lot is accepted.

5.1.5. WDwALSP with EWMA and modified EWMA

For consumer risk $\beta = 0.25$,and $(\hat{\theta}, \frac{t_{qA}}{t^0_{qA}}, \delta_0, AF, RAR, \delta) = (1.5, 1.75, 0.8, 2.5, 3, 0.8)$ the value of (n, c) from the Table 7 is (15,6).

To create history, we simulated the previous four samples (using R Language) each of size 15 having shape parameters 1.5 and scale parameter 0.5848 taking AF = 2.5. They are shown below along with the real-life data set.

0.17, 0.17, 0.82, 0.56, 1.16, **0.16**, 0.76, **0.17, 0.08,** 1.55, **0.19**, 0.33, 1.31, 0.51, 0.82
0.21, 0.54, 0.23, **0.04, 0.16,** 1.30, 1.06, 0.34, 0.78, 0.63, **0.18**, 0.65, 1.16, 0.24, 0.41
0.61, 0.27, 0.35, 1.34, 1.67, 0.36, 0.99, 0.84, 1.02, 0.63, 0.34, 0.58, 0.95, 0.62, 0.63
1.50, 0.62, 0.35, 1.67, **0.18**, 0.67, 0.54, **0.13**, 0.39, **0.20**, 0.60, **0.22**, 0.78, **0.17**, **0.04**

0.57, **0.04**, 0.28, 0.31, 1.05, 0.60, 1.38, 0.98, 0.24, 0.30, 0.49, 0.38, 1.74, **0.16**, 0.70

The inspection of the past 4 lots results in the following number of failures from 15 items using (2) we have $\boldsymbol{d_1 = 6,\ d_2 = 4,\ d_3 = 0,\ d_4 = 6}$ while the current lot is having $\boldsymbol{d_5 = 2}$. The EWMA statistics using (12) results in values 6,4.4,0.88, 4.97, and 2.59 respectively, leading to the decision to accept the lot, since $\boldsymbol{EWMA_5 = 2.59 < c = 6}$.

Under the same set of constants, the value of (n, c) for WDwALSP with Modified EWMA from the Table 16 is (10,4). Consequently, an inspection of the first 10 items from the past 4 lots and the current lot gives $\boldsymbol{d_1 = 5,\ d_2 = 3,\ d_3 = 0,\ d_4 = 3}$ and $\boldsymbol{d_5 = 1}$. The resultant Modified EWMA statistics using (17) for each lot are respectively, 5, 4.2, 2.04, 1.61, and 1.92. The lot is accepted as $\boldsymbol{MEWMA_5 = 1.92 < c = 4}$.

6. Conclusion

In this paper, LSPs are proposed based on Weibull distribution for establishing a quantile life of a lot rather quicker than the usual inspection time, which is achieved by testing the items under constant stress calledacceleration factor.More the acceleration, the lesser the inspection times but the marginally higher the sample size.Further, usage of the previous history of the lot reduces the sample size of inspection with acceleration compared to without acceleration and lot history. Hence,under acceleration, plans with EWMA or modified EWMA are more economical. Readymade tables are given for the use of the plan in industries manufacturing durable items. Theplans have straightforward extensions for other lifetime distributions as well as other time-censoring schemes.

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Table 2: Optimum parameters for WDwA assuring 50th quantile when shape parameter $\theta = 1.5$, Acceleration Factor $(AF = 1.5, 2, 2.5)$ and Acceleration Consumer Ratio $(RA = 2, 2.5, 3)$

		AF = 1.5, RA = 2				AF = 2, RA = 2.5				AF = 2.5, RA = 3			
β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$		$\delta_0 = 0.8$		$\delta_0 = 0.6$		$\delta_0 = 0.8$		$\delta_0 = 0.6$		$\delta_0 = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c
0.25	**1.25**	146	53	106	53	152	51	118	55	160	51	114	51
	1.5	47	16	36	17	51	16	41	18	57	17	38	16
	1.75	28	9	20	9	30	9	24	10	32	9	25	10
	2.0	20	6	14	6	21	6	15	6	22	6	16	6
	2.25	17	5	12	5	15	4	13	5	19	5	13	5
0.1	**1.25**	229	80	174	84	245	79	182	82	261	80	188	81
	1.5	79	25	56	25	82	24	60	25	86	24	63	25
	1.75	44	13	34	14	48	13	34	13	51	13	38	14
	2.0	33	9	23	9	35	9	25	9	37	9	26	9
	2.25	27	7	19	7	29	7	20	7	27	6	21	7
0.05	**1.25**	291	100	214	102	312	99	227	101	325	98	240	102
	1.5	97	30	69	30	109	31	77	31	111	30	78	30
	1.75	57	16	40	16	62	16	43	16	65	16	45	16
	2.0	42	11	29	11	42	10	29	10	44	10	33	11
	2.25	33	8	23	8	35	8	25	8	37	8	26	8
0.01	**1.25**	423	142	308	144	455	141	333	145	482	142	346	144
	1.5	145	43	103	43	158	43	111	43	166	43	116	43
	1.75	83	22	58	22	90	22	65	23	95	22	66	22
	2.0	58	14	40	14	67	15	46	15	67	14	46	14
	2.25	49	11	31	10	53	11	36	11	56	11	38	11

Table 3: Optimum parameters for WDwA assuring 50th quantile when shape parameter $\theta = 2.0$, Acceleration Factor $(AF = 1.5, 2, 2.5)$ and Acceleration Consumer Ratio $(RA = 2, 2.5, 3)$

		AF = 1.5, RA = 2				AF = 2, RA = 2.5				AF = 2.5, RA = 3			
β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$		$\delta_0 = 0.8$		$\delta_0 = 0.6$		$\delta_0 = 0.8$		$\delta_0 = 0.6$		$\delta_0 = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c
0.25	**1.25**	90	29	62	31	100	29	68	31	107	29	70	30
	1.5	34	10	24	11	34	9	24	10	36	9	25	10
	1.75	18	5	14	6	20	5	15	6	22	5	16	6
	2.0	15	4	10	4	13	3	8	3	14	3	11	4
	2.25	12	3	8	3	13	3	8	3	10	2	9	3
0.1	**1.25**	147	45	96	46	160	44	108	47	175	45	112	46
	1.5	52	14	35	15	58	14	36	14	62	14	41	15
	1.75	29	7	20	8	36	8	23	8	35	7	24	8
	2.0	22	5	14	5	25	5	15	5	27	5	17	5
	2.25	19	4	12	4	21	4	13	4	23	4	14	4
0.05	**1.25**	187	56	121	57	208	56	131	56	223	56	142	57
	1.5	66	17	41	17	74	17	48	18	79	17	51	18
	1.75	39	9	24	9	44	9	27	9	47	9	29	9
	2.0	29	6	18	6	33	6	20	6	35	6	21	6
	2.25	22	4	13	4	29	5	17	5	31	5	18	5
0.01	**1.25**	272	79	177	81	307	80	195	81	326	79	205	80
	1.5	98	24	61	24	110	24	70	25	118	24	72	24
	1.75	57	12	37	13	69	13	41	13	74	13	44	13
	2.0	43	8	26	8	49	8	29	8	53	8	31	8
	2.25	36	6	21	6	40	6	24	6	44	6	26	6

Table 4: Optimum parameters for WDwA assuring 50^{th} quantile when shape parameter $\theta = 2.5$, Acceleration Factor ($AF = 1.5, 2, 2.5$) and Acceleration Consumer Ratio ($RA = 2, 2.5, 3$)

		AF = 1.5, RA = 2				AF = 2, RA = 2.5				AF = 2.5, RA = 3			
β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$		$\delta_0 = 0.8$		$\delta_0 = 0.6$		$\delta_0 = 0.8$		$\delta_0 = 0.6$		$\delta_0 = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c
0.25	**1.25**	66	19	40	20	72	18	45	20	83	19	46	19
	1.5	23	6	13	6	27	6	15	6	29	6	16	6
	1.75	13	3	7	3	15	3	8	3	16	3	11	4
	2	10	2	5	2	11	2	6	2	12	2	9	3
	2.25	10	2	5	2	11	2	6	2	12	2	7	2
0.25	**1.25**	108	29	61	29	120	28	71	30	136	29	77	30
	1.5	39	9	22	9	45	9	25	9	50	9	27	9
	1.75	25	5	14	5	29	5	15	5	31	5	17	5
	2	17	3	9	3	20	3	13	4	22	3	14	4
	2.25	13	2	9	3	15	2	11	3	17	2	12	3
0.25	**1.25**	138	36	80	37	159	36	90	37	169	35	95	36
	1.5	51	11	28	11	59	11	32	11	64	11	34	11
	1.75	32	6	17	6	37	6	20	6	41	6	21	6
	2	24	4	13	4	28	4	15	4	31	4	16	4
	2.25	20	3	11	3	23	3	12	3	26	3	13	3
0.25	**1.25**	200	50	114	51	235	51	131	52	257	51	142	52
	1.5	75	15	43	16	87	15	46	15	96	15	50	15
	1.75	48	8	25	8	56	8	29	8	61	8	32	8
	2	35	5	18	5	41	5	21	5	46	5	23	5
	2.25	31	4	16	4	36	4	18	4	40	4	20	4

Table 5: Optimum parameters for WDwAwith EWMA assuring 50^{th} quantile when shape parameter $\theta = 1.5$, Acceleration Factor ($AF = 1.5$) and Acceleration Consumer Ratio ($RA = 2.0$)

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$										$\delta_0 = 0.8$									
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	**1.25**	22	8	42	15	66	24	99	36	146	53	12	6	28	14	46	23	72	36	106	53
	1.5	6	2	15	5	24	8	35	12	47	16	9	4	13	6	17	8	28	13	36	17
	1.75	6	2	9	3	15	5	19	6	28	9	5	2	7	3	9	4	16	7	20	9
	2	6	2	7	2	10	3	13	4	20	6	5	2	5	2	7	3	12	5	14	6
	2.25	4	1	7	2	7	2	10	3	17	5	3	1	5	2	5	2	7	3	12	5
0.1	**1.25**	32	11	60	21	100	35	152	53	229	80	23	11	50	24	79	38	116	56	174	84
	1.5	13	4	22	7	35	11	53	17	79	25	9	4	16	7	27	12	38	17	56	25
	1.75	7	2	14	4	21	6	31	9	44	13	5	2	10	4	17	7	24	10	34	14
	2	7	2	11	3	15	4	22	6	33	9	5	2	8	3	10	4	18	7	23	9
	2.25	4	1	8	2	12	3	19	5	27	7	3	1	8	3	8	3	13	5	19	7
0.05	**1.25**	35	12	76	26	125	43	195	67	291	100	29	14	59	28	96	46	147	70	214	102
	1.5	13	4	26	8	42	13	65	20	97	30	12	5	21	9	30	13	46	20	69	30
	1.75	7	2	18	5	25	7	39	11	57	16	5	2	10	4	20	8	30	12	40	16
	2	7	2	12	3	19	5	27	7	42	11	5	2	8	3	13	5	19	7	29	11
	2.25	4	1	12	3	16	4	21	5	33	8	3	1	8	3	11	4	17	6	23	8
0.01	**1.25**	54	18	110	37	185	62	283	95	423	142	41	19	77	36	137	64	212	99	308	144
	1.5	17	5	37	11	64	19	98	29	145	43	12	5	29	12	48	20	67	28	103	43
	1.75	11	3	23	6	38	10	57	15	83	22	8	3	16	6	26	10	42	16	58	22
	2	8	2	17	4	25	6	41	10	58	14	6	2	14	5	20	7	29	10	40	14
	2.25	8	2	13	3	22	5	32	7	49	11	6	2	9	3	15	5	22	7	31	10

Table 6: Optimum parameters for WDwAwith EWMA assuring 50^{th} quantile when shape parameter $\theta = 1.5$, Acceleration Factor $(AF = 2.0)$ and Acceleration Consumer Ratio $(RA = 2.5)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$										$\delta_0 = 0.8$									
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	18	6	42	14	69	23	104	35	152	51	15	7	30	14	54	25	77	36	118	55
	1.5	10	3	16	5	25	8	35	11	51	16	7	3	14	6	18	8	25	11	41	18
	1.75	7	2	10	3	17	5	20	6	30	9	5	2	7	3	12	5	17	7	24	10
	2	4	1	7	2	11	3	14	4	21	6	5	2	5	2	8	3	10	4	15	6
	2.25	4	1	7	2	11	3	11	3	15	4	3	1	5	2	5	2	8	3	13	5
0.1	1.25	31	10	62	20	112	36	164	53	245	79	27	12	49	22	80	36	122	55	182	82
	1.5	14	4	24	7	38	11	58	17	82	24	10	4	17	7	29	12	41	17	60	25
	1.75	11	3	15	4	22	6	33	9	48	13	5	2	13	5	18	7	26	10	34	13
	2	4	1	12	3	16	4	24	6	35	9	5	2	8	3	11	4	17	6	25	9
	2.25	4	1	8	2	13	3	17	4	29	7	3	1	6	2	9	3	14	5	20	7
0.05	1.25	38	12	82	26	139	44	208	66	312	99	27	12	61	27	99	44	153	68	227	101
	1.5	14	4	28	8	49	14	74	21	109	31	10	4	20	8	35	14	52	21	77	31
	1.75	11	3	19	5	27	7	42	11	62	16	8	3	13	5	19	7	30	11	43	16
	2	8	2	13	3	21	5	29	7	42	10	6	2	9	3	15	5	23	8	29	10
	2.25	8	2	9	2	17	4	26	6	35	8	3	1	9	3	12	4	18	6	25	8
0.01	1.25	55	17	116	36	200	62	310	96	455	141	39	17	85	37	147	64	223	97	333	145
	1.5	22	6	44	12	70	19	106	29	158	43	18	7	31	12	49	19	75	29	111	43
	1.75	12	3	25	6	41	10	62	15	90	22	11	4	17	6	31	11	43	15	65	23
	2	9	2	18	4	31	7	45	10	67	15	6	2	15	5	21	7	31	10	46	15
	2.25	9	2	15	3	24	5	34	7	53	11	6	2	10	3	17	5	26	8	36	11

Table 7: Optimum parameters for WDwAwith EWMA assuring 50^{th} quantile when shape parameter $\theta = 1.5$, Acceleration Factor $(AF = 2.5)$ and Acceleration Consumer Ratio $(RA = 3.0)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$										$\delta_0 = 0.8$									
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	22	7	44	14	69	22	110	35	160	51	18	8	34	15	54	24	76	34	114	51
	1.5	10	3	17	5	27	8	37	11	57	17	7	3	12	5	19	8	26	11	38	16
	1.75	7	2	11	3	14	4	21	6	32	9	5	2	8	3	10	4	15	6	25	10
	2	4	1	8	2	11	3	15	4	22	6	5	2	5	2	8	3	13	5	16	6
	2.25	4	1	8	2	8	2	12	3	19	5	3	1	5	2	8	3	8	3	13	5
0.1	1.25	36	11	72	22	114	35	173	53	261	80	28	12	51	22	81	35	125	54	188	81
	1.5	11	3	25	7	40	11	61	17	86	24	10	4	18	7	28	11	43	17	63	25
	1.75	8	2	16	4	23	6	35	9	51	13	8	3	11	4	19	7	27	10	38	14
	2	8	2	12	3	17	4	25	6	37	9	6	2	9	3	12	4	20	7	26	9
	2.25	5	1	9	2	13	3	18	4	27	6	3	1	6	2	9	3	15	5	21	7
0.05	1.25	40	12	83	25	146	44	219	66	325	98	33	14	61	26	106	45	160	68	240	102
	1.5	15	4	30	8	48	13	74	20	111	30	13	5	21	8	36	14	52	20	78	30
	1.75	8	2	20	5	32	8	45	11	65	16	8	3	14	5	20	7	31	11	45	16
	2	8	2	13	3	22	5	31	7	44	10	6	2	9	3	15	5	24	8	33	11
	2.25	5	1	13	3	18	4	24	5	37	8	6	2	9	3	13	4	19	6	26	8
0.01	1.25	58	17	122	36	207	61	326	96	482	142	41	17	89	37	149	62	231	96	346	144
	1.5	23	6	46	12	73	19	112	29	166	43	16	6	32	12	54	20	81	30	116	43
	1.75	13	3	26	6	43	10	65	15	95	22	9	3	18	6	30	10	45	15	66	22
	2	9	2	19	4	33	7	47	10	67	14	9	3	13	4	23	7	33	10	46	14
	2.25	9	2	15	3	25	5	36	7	56	11	7	2	11	3	17	5	25	7	38	11

Table 8: Optimum parameters for WDwAwith EWMA assuring 50^{th} quantile when shape parameter $\theta = 1.5$, Acceleration Factor $(AF = 2.5)$ and Acceleration Consumer Ratio $(RA = 3.0)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$										$\delta_0 = 0.8$									
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	16	5	25	8	43	14	62	20	90	29	8	4	16	8	26	13	42	21	62	31
	1.5	7	2	10	3	17	5	24	7	34	10	7	3	9	4	11	5	15	7	24	11
	1.75	4	1	7	2	11	3	14	4	18	5	5	2	5	2	7	3	9	4	14	6
	2	4	1	4	1	8	2	11	3	15	4	3	1	5	2	5	2	7	3	10	4
	2.25	4	1	4	1	4	1	8	2	12	3	3	1	3	1	5	2	5	2	8	3
0.1	1.25	23	7	39	12	62	19	98	30	147	45	15	7	25	12	44	21	67	32	96	46
	1.5	11	3	15	4	26	7	37	10	52	14	7	3	12	5	14	6	23	10	35	15
	1.75	4	1	12	3	16	4	21	5	29	7	5	2	5	2	10	4	13	5	20	8
	2	4	1	9	2	9	2	17	4	22	5	3	1	5	2	8	3	11	4	14	5
	2.25	4	1	5	1	9	2	14	3	19	4	3	1	3	1	6	2	9	3	12	4
0.05	1.25	27	8	50	15	80	24	127	38	187	56	15	7	32	15	53	25	83	39	121	57
	1.5	11	3	19	5	31	8	46	12	66	17	7	3	12	5	19	8	29	12	41	17
	1.75	8	2	13	3	18	4	26	6	39	9	5	2	8	3	11	4	16	6	24	9
	2	5	1	9	2	14	3	19	4	29	6	3	1	6	2	9	3	12	4	18	6
	2.25	5	1	9	2	11	2	16	3	22	4	3	1	6	2	9	3	10	3	13	4
0.01	1.25	31	9	72	21	117	34	186	54	272	79	22	10	46	21	79	36	118	54	177	81
	1.5	12	3	28	7	41	10	66	16	98	24	10	4	18	7	28	11	43	17	61	24
	1.75	9	2	18	4	28	6	38	8	57	12	6	2	11	4	17	6	23	8	37	13
	2	9	2	11	2	21	4	31	6	43	8	6	2	9	3	13	4	19	6	26	8
	2.25	6	1	11	2	17	3	24	4	36	6	4	1	7	2	10	3	14	4	21	6

Table 9: Optimum parameters for WDwAwith EWMA assuring 50th quantile when shape parameter $\theta = 2.0$, Acceleration Factor $(AF = 2.0)$ and Acceleration Consumer Ratio $(RA = 2.5)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$										$\delta_0 = 0.8$									
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	14	4	28	8	45	13	69	20	100	29	11	5	20	9	31	14	46	21	68	31
	1.5	8	2	11	3	15	4	26	7	34	9	5	2	7	3	12	5	17	7	24	10
	1.75	4	1	8	2	12	3	16	4	20	5	3	1	5	2	5	2	10	4	15	6
	2	4	1	5	1	9	2	13	3	13	3	3	1	3	1	5	2	8	3	8	3
	2.25	4	1	5	1	9	2	9	2	13	3	3	1	3	1	5	2	6	2	8	3
0.1	1.25	22	6	40	11	69	19	109	30	160	44	14	6	30	13	46	20	71	31	108	47
	1.5	8	2	17	4	25	6	41	10	58	14	5	2	13	5	18	7	26	10	36	14
	1.75	5	1	9	2	14	3	23	5	36	8	3	1	8	3	11	4	14	5	23	8
	2	5	1	9	2	14	3	19	4	25	5	3	1	6	2	9	3	12	4	15	5
	2.25	5	1	6	1	11	2	16	3	21	4	3	1	6	2	7	2	10	3	13	4
0.05	1.25	26	7	52	14	93	25	141	38	208	56	19	8	35	15	61	26	89	38	131	56
	1.5	13	3	21	5	34	8	52	12	74	17	8	3	13	5	21	8	32	12	48	18
	1.75	5	1	14	3	20	4	30	6	44	9	3	1	9	3	12	4	18	6	27	9
	2	5	1	10	2	16	3	22	4	33	6	3	1	7	2	10	3	13	4	20	6
	2.25	5	1	10	2	12	2	18	3	29	5	3	1	7	2	7	2	11	3	17	5
0.01	1.25	38	10	77	20	134	35	207	54	307	80	24	10	53	22	84	35	130	54	195	81
	1.5	14	3	32	7	46	10	74	16	110	24	11	4	20	7	31	11	45	16	70	25
	1.75	10	2	20	4	31	6	43	8	69	13	6	2	13	4	19	6	29	9	41	13
	2	6	1	17	3	23	4	35	6	49	8	6	2	10	3	14	4	21	6	29	8
	2.25	6	1	13	2	19	3	27	4	40	6	4	1	8	2	12	3	16	4	24	6

Table 10: Optimum parameters for WDwAwith EWMA assuring 50th quantile when shape parameter $\theta = 2.0$, Acceleration Factor $(AF = 2.5)$ and Acceleration Consumer Ratio $(RA = 3.0)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$										$\delta_0 = 0.8$									
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	15	4	30	8	48	13	70	19	107	29	12	5	21	9	33	14	49	21	70	30
	1.5	4	1	12	3	16	4	24	6	36	9	5	2	8	3	10	4	15	6	25	10
	1.75	4	1	9	2	13	3	13	3	22	5	3	1	6	2	8	3	11	4	16	6
	2	4	1	5	1	9	2	13	3	14	3	3	1	3	1	6	2	9	3	11	4
	2.25	4	1	5	1	5	1	10	2	10	2	3	1	3	1	3	1	6	2	9	3
0.1	1.25	23	6	47	12	74	19	113	29	175	45	17	7	32	13	51	21	78	32	112	46
	1.5	9	2	18	4	27	6	40	9	62	14	8	3	11	4	19	7	27	10	41	15
	1.75	5	1	10	2	15	3	25	5	35	7	3	1	6	2	12	4	15	5	24	8
	2	5	1	10	2	11	2	21	4	27	5	3	1	6	2	10	3	10	3	17	5
	2.25	5	1	6	1	11	2	17	3	23	4	3	1	6	2	7	2	10	3	14	4
0.05	1.25	28	7	56	14	96	24	151	38	223	56	20	8	40	16	62	25	92	37	142	57
	1.5	13	3	23	5	37	8	55	12	79	17	8	3	14	5	23	8	34	12	51	18
	1.75	10	2	15	3	21	4	32	6	47	9	6	2	9	3	13	4	19	6	29	9
	2	6	1	11	2	17	3	24	4	35	6	4	1	7	2	10	3	14	4	21	6
	2.25	6	1	11	2	13	2	19	3	31	5	4	1	7	2	8	2	12	3	18	5
0.01	1.25	37	9	86	21	144	35	222	54	326	79	23	9	54	21	92	36	138	54	205	80
	1.5	15	3	34	7	54	11	79	16	118	24	9	3	18	6	33	11	48	16	72	24
	1.75	11	2	22	4	34	6	51	9	74	13	7	2	13	4	20	6	31	9	44	13
	2	11	2	18	3	25	4	38	6	53	8	4	1	11	3	15	4	23	6	31	8
	2.25	7	1	14	2	21	3	29	4	44	6	4	1	8	2	12	3	17	4	26	6

Table 11: Optimum parameters for WDwAwith EWMA assuring 50th quantile when shape parameter $\theta = 2.5$, Acceleration Factor $(AF = 1.5)$ and Acceleration Consumer Ratio $(RA = 2.0)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$										$\delta_0 = 0.8$									
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	11	3	21	6	28	8	45	13	66	19	6	3	10	5	18	9	28	14	40	20
	1.5	4	1	8	2	12	3	16	4	23	6	4	2	7	3	9	4	11	5	13	6
	1.75	4	1	8	2	8	2	12	3	13	3	3	1	5	2	5	2	7	3	7	3
	2	4	1	5	1	5	1	9	2	10	2	3	1	3	1	5	2	5	2	5	2
	2.25	4	1	5	1	5	1	5	1	10	2	3	1	3	1	3	1	5	2	5	2
0.1	1.25	15	4	30	8	49	13	71	19	108	29	11	5	17	8	32	15	42	20	61	29
	1.5	8	2	13	3	18	4	26	6	39	9	5	2	7	3	12	5	17	7	22	9
	1.75	5	1	9	2	10	2	15	3	25	5	3	1	5	2	8	3	11	4	14	5
	2	5	1	6	1	10	2	12	2	17	3	3	1	3	1	6	2	6	2	9	3
	2.25	5	1	6	1	6	1	12	2	13	2	3	1	3	1	6	2	6	2	9	3
0.05	1.25	19	5	38	10	61	16	92	24	138	36	11	5	24	11	37	17	54	25	80	37
	1.5	9	2	14	3	23	5	36	8	51	11	5	2	10	4	15	6	20	8	28	11
	1.75	5	1	10	2	16	3	22	4	32	6	3	1	6	2	9	3	12	4	17	6
	2	5	1	6	1	12	2	18	3	24	4	3	1	6	2	6	2	9	3	13	4
	2.25	5	1	6	1	12	2	14	2	20	3	3	1	4	1	6	2	7	2	11	3
0.01	1.25	24	6	52	13	88	22	136	34	200	50	18	8	29	13	49	22	76	34	114	51
	1.5	10	2	20	4	35	7	50	10	75	15	8	3	13	5	19	7	27	10	43	16
	1.75	6	1	12	2	23	4	35	6	48	8	3	1	9	3	12	4	16	5	25	8
	2	6	1	12	2	19	3	27	4	35	5	3	1	7	2	10	3	14	4	18	5
	2.25	6	1	8	1	15	2	22	3	31	4	3	1	4	1	8	2	12	3	16	4

Table 12: Optimum parameters for WDwAwith EWMA assuring 50^{th} quantile when shape parameter $\theta = 2.5$, Acceleration Factor $(AF = 2.0)$ and Acceleration Consumer Ratio $(RA = 2.5)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$										$\delta_0 = 0.8$									
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	8	2	20	5	32	8	48	12	72	18	7	3	16	7	20	9	29	13	45	20
	1.5	5	1	9	2	13	3	18	4	27	6	5	2	5	2	10	4	10	4	15	6
	1.75	5	1	5	1	10	2	10	2	15	3	3	1	3	1	5	2	8	3	8	3
	2	5	1	5	1	6	1	10	2	11	2	3	1	3	1	3	1	6	2	6	2
	2.25	5	1	5	1	6	1	6	1	11	2	3	1	3	1	3	1	3	1	6	2
0.1	1.25	17	4	30	7	56	13	86	20	120	28	12	5	19	8	31	13	50	21	71	30
	1.5	5	1	15	3	20	4	30	6	45	9	5	2	8	3	11	4	19	7	25	9
	1.75	5	1	11	2	12	2	18	3	29	5	3	1	6	2	9	3	12	4	15	5
	2	5	1	6	1	12	2	13	2	20	3	3	1	4	1	7	2	7	2	13	4
	2.25	5	1	6	1	7	1	13	2	15	2	3	1	4	1	4	1	7	2	11	3
0.05	1.25	22	5	44	10	71	16	106	24	159	36	12	5	27	11	39	16	61	25	90	37
	1.5	10	2	16	3	27	5	42	8	59	11	6	2	11	4	17	6	23	8	32	11
	1.75	6	1	12	2	18	3	25	4	37	6	3	1	7	2	10	3	13	4	20	6
	2	6	1	7	1	14	2	20	3	28	4	3	1	4	1	7	2	11	3	15	4
	2.25	6	1	7	1	14	2	16	2	23	3	3	1	4	1	7	2	8	2	12	3
0.01	1.25	32	7	60	13	101	22	157	34	235	51	18	7	35	14	58	23	88	35	131	52
	1.5	12	2	23	4	40	7	58	10	87	15	6	2	15	5	21	7	33	11	46	15
	1.75	7	1	14	2	27	4	40	6	56	8	6	2	10	3	14	4	21	6	29	8
	2	7	1	14	2	22	3	31	4	41	5	4	1	8	2	12	3	16	4	21	5
	2.25	7	1	9	1	17	2	26	3	36	4	4	1	5	1	9	2	13	3	18	4

Table 13: Optimum parameters for WDwAwith EWMA assuring 50^{th} quantile when shape parameter $\theta = 2.5$, Acceleration Factor $(AF = 2.5)$ and Acceleration Consumer Ratio $(RA = 3.0)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$										$\delta_0 = 0.8$									
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	13	3	22	5	35	8	57	13	83	19	10	4	15	6	22	9	34	14	46	19
	1.5	5	1	10	2	15	3	20	4	29	6	3	1	8	3	8	3	13	5	16	6
	1.75	5	1	6	1	10	2	11	2	16	3	3	1	3	1	6	2	9	3	11	4
	2	5	1	6	1	6	1	11	2	12	2	3	1	3	1	3	1	6	2	9	3
	2.25	5	1	6	1	6	1	7	1	12	2	3	1	3	1	3	1	6	2	7	2
0.1	1.25	19	4	37	8	61	13	89	19	136	29	13	5	23	9	36	14	54	21	77	30
	1.5	10	2	16	3	22	4	33	6	50	9	3	1	9	3	12	4	18	6	27	9
	1.75	6	1	12	2	13	2	24	4	31	5	3	1	6	2	7	2	10	3	17	5
	2	6	1	7	1	13	2	15	2	22	3	3	1	4	1	7	2	10	3	14	4
	2.25	6	1	7	1	8	1	15	2	17	2	3	1	4	1	4	1	8	2	12	3
0.05	1.25	24	5	48	10	77	16	116	24	169	35	13	5	26	10	42	16	66	25	95	36
	1.5	11	2	18	3	29	5	46	8	64	11	6	2	12	4	16	5	25	8	34	11
	1.75	7	1	13	2	20	3	27	4	41	6	4	1	7	2	11	3	14	4	21	6
	2	7	1	8	1	15	2	22	3	31	4	4	1	4	1	8	2	12	3	16	4
	2.25	7	1	8	1	15	2	17	2	26	3	4	1	4	1	8	2	9	2	13	3
0.01	1.25	30	6	70	14	111	22	171	34	257	51	19	7	38	14	60	22	93	34	142	52
	1.5	13	2	25	4	44	7	64	10	96	15	9	3	16	5	23	7	36	11	50	15
	1.75	8	1	21	3	29	4	44	6	61	8	4	1	8	2	15	4	20	5	32	8
	2	8	1	16	2	24	3	34	4	46	5	4	1	8	2	13	3	17	4	23	5
	2.25	8	1	10	1	19	2	29	3	40	4	4	1	5	1	10	2	15	3	20	4

Table 14: Optimum parameters for WDwAwith Modified EWMA assuring 50^{th} quantile when shape parameter $\theta = 1.5$, Acceleration Factor $(AF = 1.5)$ and Acceleration Consumer Ratio $(RA = 2)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$								$\delta_0 = 0.8$							
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	22	8	33	12	44	16	58	21	12	6	22	11	32	16	44	22
	1.5	6	2	12	4	18	6	21	7	9	4	11	5	13	6	17	8
	1.75	6	2	9	3	12	4	13	4	5	2	7	3	9	4	9	4
	2	6	2	7	2	10	3	10	3	5	2	5	2	7	3	7	3
	2.25	4	1	7	2	7	2	7	2	3	1	5	2	5	2	5	2
0.1	1.25	26	9	49	17	75	26	97	34	23	11	37	18	56	27	72	35
	1.5	12	4	19	6	25	8	35	11	9	4	16	7	18	8	25	11
	1.75	7	2	10	3	17	5	17	5	5	2	10	4	12	5	12	5
	2	7	2	10	3	11	3	15	4	5	2	5	2	8	3	10	4
	2.25	4	1	8	2	8	2	12	3	3	1	5	2	8	3	8	3
0.05	1.25	32	11	61	21	90	31	122	42	23	11	44	21	65	31	86	41
	1.5	13	4	23	7	32	10	39	12	9	4	16	7	23	10	30	13
	1.75	7	2	14	4	18	5	25	7	5	2	10	4	15	6	20	8
	2	7	2	11	3	15	4	19	5	5	2	8	3	8	3	13	5
	2.25	4	1	8	2	12	3	16	4	3	1	6	2	8	3	11	4
0.01	1.25	48	16	89	30	128	43	170	57	32	15	62	29	92	43	124	58
	1.5	17	5	34	10	44	13	61	18	12	5	24	10	31	13	43	18
	1.75	11	3	19	5	30	8	34	9	8	3	13	5	21	8	26	10
	2	8	2	16	4	20	5	25	6	6	2	11	4	14	5	17	6
	2.25	8	2	13	3	17	4	22	5	6	2	9	3	12	4	15	5

Table 15: Optimum parameters for WDwAwith Modified EWMA assuring 50th quantile when shape parameter $\theta = 1.5$, Acceleration Factor ($AF = 2.0$) and Acceleration Consumer Ratio ($RA = 2.5$)

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$								$\delta_0 = 0.8$							
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	18	6	33	11	51	17	66	22	13	6	26	12	39	18	47	22
	1.5	10	3	13	4	16	5	22	7	7	3	9	4	14	6	16	7
	1.75	7	2	7	2	10	3	14	4	5	2	7	3	7	3	12	5
	2	4	1	7	2	7	2	11	3	5	2	5	2	5	2	8	3
	2.25	4	1	4	1	7	2	8	2	3	1	5	2	5	2	5	2
0.1	1.25	28	9	53	17	81	26	99	32	20	9	40	18	60	27	73	33
	1.5	10	3	17	5	27	8	34	10	10	4	12	5	22	9	24	10
	1.75	7	2	11	3	15	4	22	6	5	2	8	3	13	5	16	6
	2	4	1	8	2	12	3	16	4	5	2	8	3	8	3	11	4
	2.25	4	1	8	2	12	3	12	3	3	1	6	2	6	2	9	3
0.05	1.25	38	12	63	20	98	31	126	40	27	12	52	23	72	32	92	41
	1.5	14	4	25	7	35	10	46	13	10	4	20	8	25	10	32	13
	1.75	8	2	15	4	23	6	27	7	8	3	11	4	16	6	19	7
	2	8	2	12	3	16	4	20	5	6	2	9	3	11	4	14	5
	2.25	8	2	9	2	13	3	17	4	3	1	6	2	9	3	12	4
0.01	1.25	52	16	97	30	142	44	184	57	39	17	69	30	101	44	133	58
	1.5	18	5	33	9	51	14	66	18	13	5	23	9	36	14	46	18
	1.75	12	3	20	5	29	7	37	9	11	4	14	5	20	7	28	10
	2	9	2	17	4	22	5	27	6	6	2	12	4	15	5	19	6
	2.25	9	2	14	3	19	4	24	5	6	2	10	3	13	4	16	5

Table 16: Optimum parameters for WDwAwith Modified EWMA assuring 50th quantile when shape parameter $\theta = 1.5$, Acceleration Factor ($AF = 2.5$) and Acceleration Consumer Ratio ($RA = 3.0$)

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$								$\delta_0 = 0.8$							
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	19	6	38	12	50	16	66	21	16	7	27	12	38	17	47	21
	1.5	7	2	14	4	20	6	27	8	7	3	12	5	14	6	19	8
	1.75	7	2	7	2	11	3	14	4	5	2	5	2	10	4	10	4
	2	4	1	7	2	8	2	11	3	3	1	5	2	8	3	8	3
	2.25	4	1	4	1	8	2	8	2	3	1	5	2	6	2	8	3
0.1	1.25	33	10	59	18	85	26	108	33	21	9	42	18	58	25	79	34
	1.5	11	3	18	5	29	8	36	10	10	4	15	6	20	8	28	11
	1.75	8	2	12	3	19	5	23	6	8	3	8	3	11	4	16	6
	2	8	2	12	3	13	3	17	4	3	1	6	2	9	3	12	4
	2.25	5	1	9	2	9	2	13	3	3	1	6	2	6	2	9	3
0.05	1.25	40	12	73	22	103	31	136	41	26	11	52	22	73	31	99	42
	1.5	15	4	26	7	37	10	48	13	13	5	18	7	26	10	34	13
	1.75	8	2	16	4	24	6	28	7	8	3	11	4	14	5	20	7
	2	8	2	13	3	17	4	22	5	6	2	9	3	12	4	15	5
	2.25	5	1	9	2	14	3	18	4	6	2	7	2	10	3	13	4
0.01	1.25	51	15	102	30	146	43	197	58	36	15	72	30	108	45	137	57
	1.5	19	5	35	9	54	14	69	18	16	6	27	10	38	14	46	17
	1.75	13	3	22	5	30	7	39	9	9	3	15	5	21	7	27	9
	2	9	2	18	4	23	5	29	6	7	2	10	3	16	5	20	6
	2.25	9	2	15	3	20	4	25	5	7	2	10	3	14	4	17	5

Table 17: Optimum parameters for WDwAwith Modified EWMA assuring 50th quantile when shape parameter $\theta = 2.0$, Acceleration Factor ($AF = 1.5$) and Acceleration Consumer Ratio ($RA = 2$)

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$								$\delta_0 = 0.8$							
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	16	5	22	7	28	9	37	12	8	4	14	7	20	10	26	13
	1.5	7	2	7	2	10	3	14	4	6	3	7	3	9	4	11	5
	1.75	4	1	7	2	7	2	11	3	5	2	5	2	5	2	7	3
	2	4	1	4	1	4	1	8	2	3	1	5	2	5	2	5	2
	2.25	4	1	4	1	4	1	4	1	3	1	3	1	3	1	5	2
0.1	1.25	20	6	33	10	49	15	62	19	15	7	21	10	35	17	42	20
	1.5	8	2	11	3	18	5	22	6	7	3	7	3	12	5	14	6
	1.75	4	1	8	2	12	3	16	4	5	2	5	2	8	3	10	4
	2	4	1	8	2	9	2	9	2	3	1	5	2	6	2	8	3
	2.25	4	1	5	1	5	1	9	2	3	1	3	1	3	1	6	2
0.05	1.25	20	6	40	12	60	18	77	23	15	7	30	14	42	20	49	23
	1.5	8	2	15	4	23	6	27	7	7	3	12	5	17	7	17	7
	1.75	8	2	9	2	13	3	17	4	5	2	8	3	8	3	11	4
	2	5	1	9	2	10	2	14	3	3	1	6	2	6	2	9	3
	2.25	5	1	5	1	10	2	10	2	3	1	3	1	6	2	6	2
0.01	1.25	31	9	55	16	86	25	110	32	22	10	37	17	57	26	72	33
	1.5	12	3	24	6	32	8	41	10	10	4	15	6	20	8	28	11
	1.75	9	2	14	3	19	4	24	5	6	2	9	3	14	5	17	6
	2	5	1	10	2	15	3	20	4	3	1	6	2	9	3	12	4
	2.25	5	1	10	2	12	2	17	3	3	1	6	2	7	2	10	3

Table 18: Optimum parameters for WDwAwith Modified EWMA assuring 50th quantile when shape parameter $\theta = 2.0$, Acceleration Factor $(AF = 2)$ and Acceleration Consumer Ratio $(RA = 2.5)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$								$\delta_0 = 0.8$							
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	14	4	24	7	31	9	45	13	9	4	18	8	22	10	31	14
	1.5	8	2	11	3	15	4	15	4	5	2	7	3	7	3	12	5
	1.75	4	1	4	1	8	2	8	2	3	1	5	2	5	2	5	2
	2	4	1	4	1	8	2	8	2	3	1	3	1	5	2	5	2
	2.25	4	1	4	1	5	1	5	1	3	1	3	1	3	1	3	1
0.1	1.25	18	5	36	10	51	14	69	19	14	6	23	10	32	14	46	20
	1.5	8	2	16	4	21	5	25	6	5	2	10	4	13	5	18	7
	1.75	5	1	9	2	13	3	14	3	3	1	6	2	8	3	11	4
	2	5	1	5	1	10	2	10	2	3	1	3	1	6	2	6	2
	2.25	5	1	5	1	10	2	10	2	3	1	3	1	6	2	6	2
0.05	1.25	26	7	45	12	67	18	86	23	14	6	28	12	42	18	56	24
	1.5	12	3	17	4	26	6	30	7	8	3	11	4	16	6	21	8
	1.75	5	1	10	2	15	3	19	4	3	1	6	2	9	3	12	4
	2	5	1	10	2	11	2	16	3	3	1	6	2	7	2	10	3
	2.25	5	1	6	1	11	2	12	2	3	1	6	2	7	2	7	2
0.01	1.25	38	10	65	17	96	25	123	32	24	10	41	17	60	25	77	32
	1.5	14	3	23	5	36	8	46	10	11	4	14	5	22	8	28	10
	1.75	10	2	16	3	21	4	27	5	6	2	10	3	13	4	16	5
	2	6	1	12	2	17	3	23	4	6	2	7	2	11	3	14	4
	2.25	6	1	12	2	13	2	19	3	4	1	7	2	8	2	11	3

Table 19: Optimum parameters for WDwAwith Modified EWMA assuring 50th quantile when shape parameter $\theta = 2.0$, Acceleration Factor $(AF = 2.5)$ and Acceleration Consumer Ratio $(RA = 3)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$								$\delta_0 = 0.8$							
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	15	4	26	7	37	10	48	13	7	3	14	6	21	9	28	12
	1.5	4	1	8	2	12	3	16	4	5	2	5	2	10	4	10	4
	1.75	4	1	8	2	9	2	9	2	3	1	5	2	6	2	8	3
	2	4	1	5	1	5	1	9	2	3	1	3	1	3	1	6	2
	2.25	4	1	5	1	5	1	5	1	3	1	3	1	3	1	3	1
0.1	1.25	23	6	35	9	55	14	70	18	15	6	27	11	34	14	49	20
	1.5	9	2	17	4	22	5	27	6	8	3	11	4	14	5	19	7
	1.75	5	1	10	2	14	3	15	3	3	1	6	2	9	3	9	3
	2	5	1	10	2	11	2	11	2	3	1	6	2	7	2	9	3
	2.25	5	1	6	1	6	1	11	2	3	1	4	1	7	2	7	2
0.05	1.25	24	6	48	12	68	17	92	23	15	6	30	12	45	18	60	24
	1.5	9	2	18	4	27	6	33	7	8	3	14	5	17	6	20	7
	1.75	9	2	11	2	16	3	21	4	6	2	9	3	10	3	13	4
	2	6	1	11	2	12	2	17	3	4	1	7	2	7	2	10	3
	2.25	6	1	7	1	12	2	13	2	4	1	4	1	7	2	8	2
0.01	1.25	33	8	66	16	99	24	132	32	23	9	41	16	64	25	82	32
	1.5	15	3	25	5	39	8	49	10	9	3	15	5	24	8	30	10
	1.75	11	2	17	3	23	4	29	5	7	2	10	3	14	4	20	6
	2	7	1	13	2	19	3	25	4	4	1	8	2	11	3	15	4
	2.25	7	1	13	2	14	2	20	3	4	1	8	2	9	2	12	3

Table 20: Optimum parameters for WDwAwith Modified EWMA assuring 50th quantile when shape parameter $\theta = 2.5$, Acceleration Factor $(AF = 1.5)$ and Acceleration Consumer Ratio $(RA = 2)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$								$\delta_0 = 0.8$							
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	1.25	7	2	14	4	21	6	28	8	4	2	8	4	12	6	16	8
	1.5	4	1	8	2	8	2	12	3	4	2	5	2	7	3	7	3
	1.75	4	1	4	1	8	2	8	2	3	1	5	2	5	2	5	2
	2	4	1	4	1	5	1	5	1	3	1	3	1	3	1	5	2
	2.25	4	1	4	1	5	1	5	1	3	1	3	1	3	1	3	1
0.1	1.25	15	4	26	7	37	10	45	12	11	5	15	7	19	9	30	14
	1.5	8	2	9	2	13	3	18	4	5	2	5	2	10	4	10	4
	1.75	5	1	5	1	10	2	10	2	3	1	5	2	5	2	8	3
	2	5	1	5	1	6	1	10	2	3	1	3	1	3	1	6	2
	2.25	5	1	5	1	6	1	6	1	3	1	3	1	3	1	6	2
0.05	1.25	19	5	31	8	42	11	58	15	11	5	21	10	26	12	37	17
	1.5	9	2	13	3	18	4	23	5	5	2	8	3	10	4	13	5
	1.75	5	1	10	2	11	2	15	3	3	1	6	2	6	2	9	3
	2	5	1	6	1	11	2	12	2	3	1	3	1	6	2	6	2
	2.25	5	1	6	1	7	1	7	1	3	1	3	1	4	1	6	2
0.01	1.25	20	5	40	10	60	15	80	20	16	7	27	12	38	17	47	21
	1.5	10	2	15	3	25	5	30	6	8	3	11	4	16	6	19	7
	1.75	6	1	12	2	17	3	23	4	3	1	6	2	9	3	12	4
	2	6	1	7	1	13	2	14	2	3	1	6	2	7	2	10	3
	2.25	6	1	7	1	13	2	14	2	3	1	4	1	7	2	8	2

Table 21: Optimum parameters for WDwAwith Modified EWMA assuring 50^{th} quantile when shape parameter $\theta = 2.5$, Acceleration Factor $(AF = 2.0)$ and Acceleration Consumer Ratio $(RA = 2.5)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$								$\delta_0 = 0.8$							
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	**1.25**	8	2	16	4	24	6	32	8	7	3	9	4	16	7	18	8
	1.5	5	1	9	2	9	2	13	3	5	2	5	2	5	2	10	4
	1.75	5	1	5	1	5	1	9	2	3	1	3	1	5	2	5	2
	2	5	1	5	1	5	1	5	1	3	1	3	1	3	1	3	1
	2.25	5	1	5	1	5	1	5	1	3	1	3	1	3	1	3	1
0.1	**1.25**	13	3	26	6	39	9	52	12	12	5	17	7	24	10	31	13
	1.5	5	1	10	2	15	3	20	4	5	2	8	3	8	3	11	4
	1.75	5	1	6	1	11	2	12	2	3	1	3	1	6	2	6	2
	2	5	1	6	1	7	1	12	2	3	1	3	1	6	2	6	2
	2.25	5	1	6	1	7	1	7	1	3	1	3	1	4	1	4	1
0.05	**1.25**	22	5	35	8	53	12	66	15	12	5	22	9	29	12	39	16
	1.5	10	2	15	3	21	4	26	5	6	2	9	3	11	4	14	5
	1.75	6	1	11	2	12	2	18	3	3	1	6	2	7	2	10	3
	2	6	1	7	1	12	2	13	2	3	1	4	1	7	2	7	2
	2.25	6	1	7	1	8	1	13	2	3	1	4	1	4	1	7	2
0.01	**1.25**	27	6	50	11	74	16	97	21	15	6	28	11	43	17	53	21
	1.5	11	2	22	4	29	5	39	7	6	2	12	4	15	5	21	7
	1.75	7	1	13	2	20	3	26	4	6	2	7	2	11	3	11	3
	2	7	1	13	2	15	2	17	2	4	1	7	2	8	2	11	3
	2.25	7	1	9	1	15	2	17	2	4	1	5	1	8	2	9	2

Table 22: Optimum parameters for WDwAwith Modified EWMA assuring 50^{th} quantile when shape parameter $\theta = 2.5$, Acceleration Factor $(AF = 2.5)$ and Acceleration Consumer Ratio $(RA = 3.0)$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$								$\delta_0 = 0.8$							
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	**1.25**	13	3	22	5	26	6	35	8	10	4	12	5	17	7	22	9
	1.5	5	1	10	2	10	2	15	3	3	1	6	2	8	3	8	3
	1.75	5	1	5	1	10	2	10	2	3	1	3	1	3	1	6	2
	2	5	1	5	1	6	1	6	1	3	1	3	1	3	1	3	1
	2.25	5	1	5	1	6	1	6	1	3	1	3	1	3	1	3	1
0.1	**1.25**	14	3	28	6	42	9	56	12	10	4	18	7	23	9	31	12
	1.5	10	2	11	2	17	3	22	4	3	1	6	2	9	3	12	4
	1.75	6	1	11	2	12	2	13	2	3	1	6	2	7	2	7	2
	2	6	1	7	1	7	1	13	2	3	1	4	1	4	1	7	2
	2.25	6	1	7	1	7	1	8	1	3	1	4	1	4	1	4	1
0.05	**1.25**	24	5	39	8	53	11	72	15	13	5	21	8	29	11	42	16
	1.5	11	2	17	3	23	4	29	5	6	2	9	3	12	4	16	5
	1.75	6	1	12	2	14	2	20	3	4	1	7	2	7	2	10	3
	2	6	1	8	1	14	2	15	2	4	1	4	1	7	2	8	2
	2.25	6	1	8	1	9	1	15	2	4	1	4	1	5	1	8	2
0.01	**1.25**	30	6	55	11	80	16	106	21	19	7	30	11	41	15	60	22
	1.5	12	2	24	4	31	5	43	7	7	2	10	3	17	5	20	6
	1.75	8	1	15	2	22	3	29	4	4	1	8	2	11	3	12	3
	2	8	1	15	2	17	2	24	3	4	1	8	2	9	2	12	3
	2.25	8	1	10	1	17	2	19	2	4	1	5	1	9	2	10	2

Table 23: Optimum parameters for the Weibull Distribution assuring 50^{th} quantile when shape parameter $\theta = 2$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$		$\delta_0 = 0.8$	
		n	c	n	c
0.25	**1.25**	20	3	15	4
	1.5	14	2	9	2
	1.75	14	2	9	2
	2	9	1	5	1
	2.25	9	1	5	1
0.1	**1.25**	38	5	22	5
	1.5	26	3	16	3
	1.75	20	2	12	2
	2	20	2	12	2
	2.25	14	1	8	1
0.05	**1.25**	49	6	29	6
	1.5	31	3	18	3
	1.75	31	3	14	2
	2	25	2	14	2
	2.25	25	2	14	2
0.01	**1.25**	74	8	43	8
	1.5	56	5	32	5
	1.75	42	3	24	3
	2	42	3	24	3
	2.25	35	2	20	2

Table 24: Optimum parameters for the Weibull Distributionwith EWMA assuring 50^{th} quantilewhen shape parameter $\theta = 2$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$										$\delta_0 = 0.8$									
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 1.0$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	**1.25**	6	1	7	1	12	2	13	2	20	3	4	1	4	1	8	2	11	3	15	4
	1.5	6	1	7	1	7	1	13	2	14	2	4	1	4	1	4	1	8	2	9	2
	1.75	6	1	7	1	7	1	8	1	14	2	4	1	4	1	4	1	5	1	9	2
	2	6	1	7	1	7	1	8	1	9	1	4	1	4	1	4	1	5	1	5	1
	2.25	6	1	7	1	7	1	8	1	9	1	4	1	4	1	4	1	5	1	5	1
0.1	**1.25**	7	1	14	2	16	2	29	4	38	5	4	1	9	2	9	2	17	4	22	5
	1.5	7	1	8	1	16	2	18	2	26	3	4	1	5	1	9	2	11	2	16	3
	1.75	7	1	8	1	10	1	18	2	20	2	4	1	5	1	6	1	11	2	12	2
	2	7	1	8	1	10	1	12	1	20	2	4	1	5	1	6	1	7	1	12	2
	2.25	7	1	8	1	10	1	12	1	14	1	4	1	5	1	6	1	7	1	8	1
0.05	**1.25**	8	1	16	2	24	3	33	4	49	6	5	1	9	2	14	3	19	4	29	6
	1.5	8	1	10	1	18	2	27	3	31	3	5	1	6	1	11	2	12	2	18	3
	1.75	8	1	10	1	12	1	21	2	31	3	5	1	6	1	7	1	12	2	14	2
	2	8	1	10	1	12	1	14	1	25	2	5	1	6	1	7	1	8	1	14	2
	2.25	8	1	10	1	12	1	14	1	25	2	5	1	6	1	7	1	8	1	14	2
0.01	**1.25**	9	1	25	3	36	4	54	6	74	8	9	2	11	2	21	4	31	6	43	8
	1.5	9	1	19	2	23	2	35	3	56	5	6	1	11	2	13	2	20	3	32	5
	1.75	9	1	13	1	23	2	28	2	42	3	6	1	7	1	13	2	16	2	24	3
	2	9	1	13	1	16	1	28	2	42	3	6	1	7	1	9	1	16	2	24	3
	2.25	9	1	13	1	16	1	28	2	35	2	6	1	7	1	9	1	16	2	20	2

Table 25: Optimum parameters for the Weibull Distributionwith Modified EWMA assuring 50^{th} quantile when shape parameter $\theta = 2.5$

β	$\frac{t_{qA}}{t^0_{qA}}$	$\delta_0 = 0.6$								$\delta_0 = 0.8$							
		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$		$\lambda = 0.2$		$\lambda = 0.4$		$\lambda = 0.6$		$\lambda = 0.8$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
0.25	**1.25**	6	1	6	1	7	1	12	2	4	1	4	1	4	1	8	2
	1.5	6	1	6	1	7	1	7	1	4	1	4	1	4	1	4	1
	1.75	6	1	6	1	7	1	7	1	4	1	4	1	4	1	4	1
	2	6	1	6	1	7	1	7	1	4	1	4	1	4	1	4	1
	2.25	6	1	6	1	7	1	7	1	4	1	4	1	4	1	4	1
0.1	**1.25**	7	1	8	1	14	2	15	2	4	1	8	2	9	2	9	2
	1.5	7	1	8	1	9	1	15	2	4	1	5	1	5	1	9	2
	1.75	7	1	8	1	9	1	10	1	4	1	5	1	5	1	6	1
	2	7	1	8	1	9	1	10	1	4	1	5	1	5	1	6	1
	2.25	7	1	8	1	9	1	10	1	4	1	5	1	5	1	6	1
0.05	**1.25**	8	1	15	2	16	2	23	3	5	1	9	2	10	2	14	3
	1.5	8	1	9	1	10	1	18	2	5	1	5	1	6	1	10	2
	1.75	8	1	9	1	10	1	12	1	5	1	5	1	6	1	7	1
	2	8	1	9	1	10	1	12	1	5	1	5	1	6	1	7	1
	2.25	8	1	9	1	10	1	12	1	5	1	5	1	6	1	7	1
0.01	**1.25**	9	1	18	2	26	3	35	4	5	1	10	2	15	3	20	4
	1.5	9	1	12	1	20	2	23	2	5	1	7	1	12	2	13	2
	1.75	9	1	12	1	14	1	23	2	5	1	7	1	8	1	13	2
	2	9	1	12	1	14	1	16	1	5	1	7	1	8	1	9	1
	2.25	9	1	12	1	14	1	16	1	5	1	7	1	8	1	9	1