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Robust Parameter Design Using 20 Run Plackett-Burman Design

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Abstract

Taguchi's parameter design technique for improving product quality has aroused a great deal of interest among statisticians and quality practitioners. He proposed the use of product array for reducing variation and improving product quality. However, in some applications, his approach results in an exorbitant number of runs. As an alternative to the product array approach, Welch *et al.* (1990), Shoemaker *et al.* (1991) and Montgomery (1991a), proposed the use of combined arrays wherein control and noise factors are combined in a single array. Further, Shoemaker *et al.* (1991) used an optimal design algorithm to reduce the size of the combined array.

In this paper, we have exploited the non-orthogonal column structure of the 20-run Plackett-Burman Design. It is shown that by making use of the columns of the 20-run Plackett-Burman design, the size of the experiment can further be reduced. The results have been shown for designs with six factors and the results for three, four and five factors are given in the annexure.

Key words: Robust parameter design; Orthogonal arrays; Fractional factorial designs; Combined array; Plackett-Burman designs; D-efficiency; Projective rationale.

1. Introduction

Taguchi (1959, 1987) introduced an off-line quality control technique known as robust parameter design for reducing variation and improving product quality. The root of this idea is the notion that products lack in quality because of inconsistency in performance produced by factors that are controllable in the design of the product. He thus classifies the factors into two groups: Control factors and Noise factors.

The overall objective of Taguchi's approach is to determine the levels of control factors at which the effect of the noise factors on the performance characteristics is minimized. To achieve this objective he made use of product arrays by taking the Kronecker product of two orthogonal arrays, one involving only the control factors (inner array) and the other involving only the noise factors (outer array). Direct products of orthogonal arrays are themselves orthogonal arrays but the product operation greatly increases the number of observations in the array without generally increasing its strength. Several different methods of construction have been suggested, with the underlying idea of choosing levels for the controllable factors so that the uncontrollable factors have least influence on the response. Welch *et al*. (1990), Borkowski and Lucas (1991, 1997), Montgomery (1991 a, b), Myers (1991), Shoemaker *et al.* (1991), Welch and Sacks (1991), Box and Jones (1992) and Lucas (1994) suggested the

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use of combined arrays, wherein the control factors and noise factors are combined in a single array. A combined array lets the experimenter choose the interactions to be estimated. This provides more flexibility so that the experimental budget can be used to fit models more refined than the main effects only models frequently used in Taguchi's loss model approach. An excellent review of the robust parameter technique is made by Nair (1992) and Myers and Montgomery (1995). Kunert *et al*. (2007) compared Taguchi's product array with a combined array.

In this paper we have exploited the non-orthogonal column structure of the 20-run Plackett-Burman (PB) design to generate non-orthogonal combined arrays.

2. The Role of Interactions

In parameter design, one is interested in choosing the levels of control factors so that the product's performance is insensitive to noise factors and can be adjusted on target as appropriate. The control × noise (C × N) interactions are exploited to accomplish this. The structure of these interactions provides special insights in the combined array/response model approach because they are the effects that can be exploited to reduce response variability. The noise × noise (N × N) interactions play little role in making a product's performance insensitive to noise factors. The presence of large C × C interactions is considered highly undesirable; thus, every attempt is made to reduce the number of C × C interactions through judicious choice of the quality characteristics.

3. Objectives and the Supportive Models

Keeping in view the above justification for the inclusion of various terms in the models we now specify our objectives:

Let there be r control factors, say, $x_1, x_2, \ldots\ldots, x_r$ and s noise factors *viz.* $z_1, z_2, \ldots\ldots, z_s$.

Then our objective is:

1. To estimate the main effects of all the control factors and noise factors.
2. To estimate C × N interactions.
3. To estimate if possible, (depending on the degrees of freedom) the C × C interactions.

The above objectives can be explained more precisely with the help of regression models. Let y denote a quality characteristic associated with a product. We can then express:

$$y = f(x, z) \tag{1}$$

If the response is well modelled by a linear function of the independent variables, then the approximating function is the first order model:

$$y = \beta_0 + \sum \beta_i x_i + \sum \gamma_j z_j + \epsilon \tag{2}$$

But, in model (2), the settings of x have no influence on variability. For robust parameter design to be successful, the functional relationship between control factors and noise variables should be such that they interact. Thus, a second order model will be more appropriate:

$$y = \beta_0 + \sum \beta_i x_i + \sum \beta_{ii} x_i^2 + \sum\sum \beta_{ii'} x_i x_{i'} + \sum \gamma_j z_j + \sum \gamma_{jj} z_j^2 + \sum\sum \gamma_{jj'} z_j z_{j'} + \sum\sum \delta_{ij} x_i z_j + \epsilon$$

where $i \neq i' = 1,2,....,r;\ j \neq j' = 1,2,....,s$ (3)

In order to meet the first two objectives mentioned above the reduced model, by keeping the origin at (0, 0), would be:

$$y = \sum \beta_i x_i + \sum \gamma_j z_j + \sum\sum \delta_{ij} x_i z_j + \epsilon. \quad (4)$$

Whereas, when one is also interested in estimating the C × C interactions (the third objective), the corresponding model would be:

$$y = \sum \beta_i x_i + \sum \gamma_j z_j + \sum\sum \delta_{ij} x_i z_j + \sum\sum \beta_{ii'} x_i x_{i'} + \epsilon \quad (5)$$

4. Efficiency Criterion

We have used the following D-criterion for measuring the overall efficiency for estimating a collection of effects:

$$\text{D-efficiency} = |X'X|^{1/k} \quad (6)$$

where, $X = [x_1/||x_1||, ..., x_k/||x_k||]$; and x_i is the coefficient vector of the i^{th} effect. To find the efficiency of each individual effect, we have used the following D_s criterion:

$$\frac{\left\{x_i' x_i - x_i' X_{(i)} \left(X_{(i)}' X_{(i)}\right)^{-1} X_{(i)}' X_{(i)}\right\}}{x_i' x_i} \quad (7)$$

where, $X_{(i)}$ is obtained from X by deleting x_i.

5. Steps Used for Combined Array Approach

We give below the steps used in the combined array approach:

i. Choose p columns from the totality of n–1 columns and consider all the non-equivalent designs.
ii. For each design allocate the control factors and noise factors to p columns.
iii. Write the appropriate model by considering the required set of C × N interactions and C × C interactions (depending upon run-size).
iv. For all possible choices of the control and noise factors find the D value for the whole design and D_s values for the various effects.
v. Compare the D value of all the designs obtained and take the one with maximum D value. If there are more designs with the maximum D value, consider all of them.
vi. Sort the D_s values of these designs on the basis of C × N interactions and take the design for which it is maximum. If there are more than one designs with the same values of D_s for C × N interactions, consider all of them.

vii. Sort the D_s values of these designs on the basis of C × C interactions and take the design for which it is minimum. If there are more than one designs with the same values of D_s for C × C interactions, take all of them.

viii. Among the designs chosen by step (vii), finally sort these designs on the basis of the D_s values for control factors and noise factors and select the design for which it is maximum.

ix. Once a design has been selected by following the aforesaid steps, the D_s values of the various effects are reported according to the order of column allocations of respective control factors, noise factors and their interactions in the tables.

6. Plackett-Burman Designs

Plackett and Burman (1946) provided a series of two-level fractional factorial designs, for examining (n–1) factors in n runs, where n is a multiple of 4 and $n \leq 100$. These are non-orthogonal designs in which the aliasing coefficient between any two effects lies between –1 and +1. They gave the following design for 20-runs:

Table 1: 20-Run Plackett-Burman design

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
–	+	–	–	+	+	+	+	–	+	–	+	–	–	–	–	+	+	–
–	–	+	–	–	+	+	+	+	–	+	–	+	–	–	–	–	+	+
+	–	–	+	–	–	+	+	+	+	–	+	–	+	–	–	–	–	+
+	+	–	–	+	–	–	+	+	+	+	–	+	–	+	–	–	–	–
–	+	+	–	–	+	–	–	+	+	+	+	–	+	–	+	–	–	–
–	–	+	+	–	–	+	–	–	+	+	+	+	–	+	–	+	–	–
–	–	–	+	+	–	–	+	–	–	+	+	+	+	–	+	–	+	–
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+	+	–	+	–	+	–	–	–	–	+	+	–	–	+	–	–	+	+
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+	+	+	+	–	+	–	+	–	–	–	–	+	+	–	–	+	–	–
–	+	+	+	+	–	+	–	+	–	–	–	–	+	+	–	–	+	–
–	–	+	+	+	+	–	+	–	+	–	–	–	–	+	+	–	–	+
+	–	–	+	+	+	+	–	+	–	+	–	–	–	–	+	+	–	–
–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–

We shall now discuss the projection properties of this design. The choice of p columns, where $p \leq (n–1)$ may result in a number of designs for given n and p, not all of which may be equivalent. Two such designs are said to be equivalent if one can be obtained from the other by permutations of rows, columns and sign changes. Draper and Lin (1990) have given detailed tables giving the number of distinct designs for 12-, 20- and 24-run PB designs for different values of p. Each design is characterized by the number of repeat runs, mirror images or distinct runs it has. For p = 2, Draper and Lin (1990, Table 3B), Lin and Draper

(1992, 1995) found that projection is a 2^2 design, $n/4$ times over. For $p = 3$, they found that there are two different projections, each one consisting of at least a full 2^3 factorial. For $p = 4$, there are three non-isomorphic 20×4 submatrices: designs 4.1, 4.2 and 4.3. The 20 points in design 4.1 have one treatment combination omitted and five duplicated. Both designs 4.2 and 4.3 have each 4 points missing, 5 points appear once, 6 points appear twice and one point appears three times. For $p = 5$, Draper and Lin (1990, Table 3B) found that there are nine non-isomorphic 20×5 submatrices *viz.* designs 5.1, 5.2, …, and 5.9. Design 5.1 has no run with repeats, design 5.4 has one run with 2 repeats and the remaining designs have at least two runs with repeats. For $p = 6$, there are 50 non-isomorphic 20×6 submatrices. To save the enormity of calculations, we consider only the 17 designs considered by Draper and Lin (1990, Table 3B) *viz.* designs 6.1, 6.2,…, and 6.17 based on their mirror image patterns or repeat run pairs. Designs 6.1, 6.2, 6.4, 6.9 and 6.13 have no runs with repeats while rest of the designs have at least one run with a repeat. We now discuss the combined array concept for this design.

7. Combined Array Results for the 20-Run PB Design

There are 19 independent columns for studying the factor effects and 20 design points. We shall discuss here only one case as others can be obtained in a similar manner. Suppose we have six factors we then need to choose six columns from the 19 columns. Now 6 factors can be divided into control and noise factors in five different ways:

(a) $r = 5, s = 1$ (b) $r = 1, s = 5$ (c) $r = 4, s = 2$ (d) $r = 2, s = 4$ (e) $r = 3, s = 3$

Consider the first possibility:

(a) $r = 5, s = 1$

Allocate five columns to the control factors and one to the noise factor. There are 21 parameters to be estimated including the C × C interactions. Out of 17 designs given by Draper and Lin (1990, Table 3B), 5 designs have no repeats and thus enable us to estimate 19 parameters in 20 runs. Out of these, design 6.1 is the best having maximum D-efficiency. The following Table gives the allocation of control and noise factors which have come out to be the best for this design:

Table 2: $r = 5, s = 1$

Design 6.1 (1,2,3,4,5,6), (20)						
D. No.	C	N	C × N	C × C	D	D_s
1	1,3,4,5,6	2	12,32,42,52,62	14,15,34,35, 36,45,46,56	.73	.31,.31,.56,.44,.64,.58,.28,.57,.64,.51,.56, .58,.41, .57,.43,.32,.51,.28,.44

In the Table after giving the design number we give the column allocation of the selected design in the first parenthesis and the number in the second parenthesis gives the number of distinct runs in the design.

Five out of 17 designs have one run with a repeat and thus enable us to estimate 18 parameters. Design 6.5 is the best. We call a design to be good if it has the highest D-efficiency and provides maximum flexibility in the allocation of control and noise factors.

We give below the allocations of control and noise factors that have come out to be the best for this design:

Table 3: $r = 5, s = 1$

Design 6.5 (1,2,4,5,6,7), (19)						
D. No.	C	N	C × N	C × C	D	D_s
1	1,2,4,6,7	5	15,25,45,65,75	12,14,16,17, 26, 27,46	.74	.68,.49,.36,.64,.64,.68,.66,.59,.56, .38,.33,.59,.56,.38,.33,.53,.33,.49
2	2,4,5,6,7	1	21,41,51,61,71	25.26,27,45, 46,56,57	.74	.49,.36,.68,.64,.64,.68,.59,.56,.66, .38,.33,.59,.53,.33,.56,.49,.38,.33

There are 4 designs with 2 repeats, out of which, design 6.10 is the best having the highest D-efficiency, which enables us to estimate 17 parameters in 18 runs. The following Table gives the allocation of control and noise factors that has come out to be the best for this design:

Table 4: $r = 5, s = 1$

Design 6.10 (1,4,5,6,7,9), (18)						
D. No.	C	N	C × N	C × C	D	D_s
1	1,4,5,6,9	7	17,47,57,67,97	14,16,45,46, 56,59	.71	.57,.77,.51,.67,.28,.67,.4,.54,.4, .67, .41,.31,.23,.49,.22,.31,.22

There are 3 designs with 3 repeats, out of which design 6.16 is the best having highest D-efficiency. This design enables us to estimate 16 parameters in 17 runs. The following table gives the allocation of control and noise factors that have come out to be the best for this design:

Table 5: $r = 5, s = 1$

Design 6.16 (1,2,3,6,9,12), (17)						
D. No.	C	N	C × N	C × C	D	D_s
1	1,2,3,6,9	12	112,212,312, 612,912	12,13,16,23, 26	.69	.28,.77,.18,.18,.10,.29,.77,.31, .54, .54,.10, .24,.05,.28,.16,.54

(b) $r = 1, s = 5$

Allocate one column to the control factor and five to the noise factors. In this case 11 parameters are to be estimated as there are no C × C interactions. Design 6.17 is the best having highest D-efficiency, which estimates all the 11 parameters in minimum number of runs. We give below the allocations of control and noise factors which have come out to be the best for this design:

Table 6: $r = 1, s = 5$

Design 6.17 (1,2,3,5,8,13), (17)						
D. No.	C	N	C × N	C × C	D	D_s
1	1	2,3,5,8,13	12,13,15,18,113	-	0.93	.93,.83,.86,.80,.86,.86,.86, .86,.81, .86,.86
2	2	1,3,5,8,13	21,23,25,28,213	-	0.93	.92,.86,.86,.86,.80,.86,.86, .86,.86, .81,.86
3	13	1,2,3,5,8	131,132,133,135,138	-	0.93	.93,.83,.86,.80,.86,.86,.86, .86,.81, .86,.86

(c) $r = 4, s = 2$

Allocate four columns to the control factors and two to the noise factors. There are in all 20 parameters to be estimated. However, in 20 runs one can estimate at the most 19 parameters. Out of 5 designs, having no repeats, design 6.1 is the best having maximum D-efficiency. The following table gives the allocation of control and noise factors which has come out to be the best for this design:

Table 7: $r = 4, s = 2$

Design 6.1 (1,2,3,4,5,6), (20)						
D. No.	C	N	C × N	C × C	D	D_s
1	1,2,3,6	4,5	14,15,24,25,34, 35,64,65	12,13,23,26,36	.68	.25,.56,.31,.13,.39,.3,.56,.46, .64, .2,.56,.47, .32, .29, .23, .13, .39, .56, .37

Out of 5 designs having one repeat, design 6.6 performs the best. We give below the allocation of control and noise factors that has come out to be the best for this design:

Table 8: $r = 4, s = 2$

Design 6.6 (1,2,4,5,7,8), (19)						
D. No.	C	N	C × N	C × C	D	D_s
1	1,2,5,8	4,7	14,17,24,27,54, 57, 84,87	12,15,25,28	.74	.53,.66,.66,.53,.38,.38,.64,.64,.36, .36,.49,.49,.56,.56,.53,.36,.66,.33

There are 4 designs with 2 repeats, out of which design 6.10 is the best having the highest D-efficiency, which enables us to estimate 17 parameters in 18 runs. The following table gives the allocation of control and noise factors that has come out to be the best for this design:

Table 9: $r = 4, s = 2$

Design 6.10 (1,4,5,6,7,9), (18)						
D. No.	C	N	C × N	C × C	D	D_s
1	1,4,5,9	6,7	16,17,46,47,56, 57,96,97	14,15,19	.71	.55,.29,.55,.42,.67,.67,.31,.32,.27, .58, .23,.32,.22,.55,.49,.67,.49

There are 3 designs with 3 repeats, out of which design 6.16 is the best having the highest D-efficiency. This design enables us to estimate 16 parameters in 17 runs. The following Table gives the allocation of control and noise factors that has come out to be the best for this design:

Table 10: $r = 4, s = 2$

Design 6.16 (1,2,3,6,9,12), (17)						
D. No.	C	N	C × N	C × C	D	D_s
1	1,3,6,9	2,12	12,112,32,312,62,612, 92,912	13,16	.63	.14,.31,.07,.23,.77,.47,.33,.24, .33,.54,.33,.54, .08,.24,.05,.28

Consider the following example discussed by Shoemaker *et al.* (1991) to illustrate the flexibility afforded by a combined array:

7.1. Example 1

Suppose there are 4 two-level control factors, *A*, *B*, *C* and *D*, and 2 two-level noise factors, *r* and *s*. Assume that the control × control interactions – *AB*, *AC* and *AD* are potentially important and that we wish to estimate them. If we use the product array approach, we first construct a control array (CA) that estimates all main effects – *A*, *B*, *C* and *D* and the three important interactions – *AB*, *AC* and *AD*. We then construct a noise array (NA) that estimates the two main effects – *r* and *s*, and the interaction – *rs*. The defining relation of this plan is I = *ABCD*. According to the general result concerning estimation capacity of CA × NA designs, the resulting 32-run product array allows us to estimate six main effects – *A*, *B*, *C*, *D*, *r*, and *s*, the 12 two-factor interactions – *AB, AC, AD, rs, Ar, Br, Cr, Dr, As, Bs, Cs*, and *Ds* and 13 higher-order interactions. On the other hand, a combined array 2^{6-1} with resolution VI using

$$\mathrm{I} = x_1x_2x_3x_4z_1z_2$$

is much more appropriate. This design allows the estimation of all the six main effects and all 15 two-factor interactions.

As yet a better approach, Shoemaker *et al*. (1991) used an optimal design algorithm to reduce the size of experiment further. As the 13 higher order interactions are less likely to be important, they constructed a linear model consisting of six main effects and 12 two-factor interactions mentioned above. Three combined arrays of size 20, 22, and 24 were generated from an optimal design algorithm DETMAX (Mitchell 1974), used in the software system RS/ DISCOVER (1988). All the three designs are approximately two-third the size of the product /combined array but allow efficient estimation of all the main effects and two-factor interactions mentioned earlier.

For the above example, we exploited the non-orthogonal column structure of the 20-run PB design. Also, as the role of noise × noise interactions in making a product's performance insensitive to noise factors is almost negligible, we therefore exclude them from our model. We are now left with 17 parameters to be estimated. There are 4 designs with 2 repeats each, *viz*, 6.8, 6.10, 6.11, and 6.14 given by Draper and Lin (1990, Table 3B). As a result, they have only seventeen degrees of freedom for estimating factor effects. Out of these four designs, design 6.10 estimates the 17 parameters with highest D-efficiency. Table 9 gives the allocation of control and noise factors that has come out to be the best for this design.

Thus, if we allocate the four control factors to columns 1, 4, 5, and 9 and noise factors to columns 6 and 7 of design 6.10, this design allows us to estimate all the 17 parameters in 18 runs only as compared to the design given by Shoemaker *et al.* (1991).

(c) $r = 2, s = 4$

Allocate two columns to the control factors and four to the noise factors. There are in all 15 parameters to be estimated. Out of 17 designs, designs 6.15, 6.16, and 6.17 enable us to estimate 15 parameters in 17 runs. However, as design 6.17 provides more flexibility for the allocation of control and noise factors, we give below the results for this design only:

Table 11: $r = 2, s = 4$

Design 6.17 (1,2,3,5,8,13), (17)						
D. No.	C	N	C × N	C × C	D	D_s
1	2,3	1,5,8,13	21, 25, 28, 213, 31, 35, 38, 313	23	.66	.35,.52,.35,.12,.25,.30,.46,.20,.24, .20, .28,.37, .06, .16, .48
2	8,13	1,2,3,5	81, 82, 83, 85, 131, 132, 133, 135	813	.66	.52,.35,.35,.30,.25,.12,.28,.16,.06, .37,.46,.20,.24,.20,.48

(d) $r = 3, s = 3$

Allocate three columns to the control factors and three to noise factors. Out of 17 designs, 5 designs have one repeat and thus enable us to estimate 18 parameters in 19 runs. Out of these 5 designs, design 6.5 is the best having maximum D value. The following table gives the allocation of control and noise factors that has come out to be the best for this design:

Table 12: $r = 3, s = 3$

Design 6.5 (1,2,4,5,6,7), (19)						
D. No.	C	N	C × N	C × C	D	D_s
1	1,2,5	4,6,7	14,16,17,24,26,27, 54,56,57	12,15,25	.68	.66,.42,.66,.36,.64,.12,.48,.19, .36,.12,.36,.26,.48,.19,.36,.42, .66,.42

Out of 4 designs having 2 repeats, design 6.10 performs the best and enables us to estimate 17 parameters in 18 runs. The following Table gives the allocation of control and noise factors that has come out to be the best for this design:

Table 13: r = 3, s = 3

Design 6.10 (1,4,5,6,7,9), (18)						
D. No.	C	N	C × N	C × C	D	D_s
1	1,6,7	4,5,9	14,15,19,64,65,69, 74,75,79	16,17	.71	.55,.67,.67,.29,.55,.42,.48,.67,.48, .27,.23,.22,.58,.32,.55,.31,.32

Out of 3 designs having 3 repeats, design 6.17 performs the best and enables us to estimate 16 parameters in 17 runs. The following Table gives the allocation of control and noise factors which have come out to be the best for this design:

Table 14: $r = 3, s = 3$

Design 6.17 (1,2,3,5,8,13), (17)						
D. No.	C	N	C × N	C × C	D	D_s
1	1,3,13	2,5,8	12,15,18,32,35,38, 132, 135,138	13	.63	.32,.53,.53,.28,.14,.28,.77,.28, .30,.28, .20,.05,.10,.12,.17,.34
2	2,3,13	1,5,8	21,25,28,31,35,38, 131, 135,138	23	.63	.32,.53,.53,.28,.28,.14,.77,.30, .28, .28,.05,.20,.10,.17,.12,.34

In the presence of 3 two-level control factors and 3 two-level noise factors, Shoemaker *et al.* (1991) have shown with the help of an example, the flexibility offered by a combined array *vis-a-vis* a product array.

8. Concluding Remarks

Many authors have advocated the use of combined arrays as an alternative to Taguchi's product arrays by modelling the response itself as a function of control and noise factors. These combined arrays are based on orthogonal fractional factorial designs, which do not exist for all values of *n*. Also, a major concern of most of the industries is to reduce the number of runs or minimize it. In this paper, we have exploited the non-orthogonal column structure of the 20-run Plackett-Burman design, giving a systematic method for choosing columns of a PB design for the allocation of control and noise factors. It has been shown that most of the designs using this approach, though not orthogonal, result in the reduction of the size of the experiment, a major benefit to the industry.

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ANNEXURE

Combined Array Designs for Three, Four and Five Factors

For $p = 3$

(a) $r = 2, s = 1$

Design 3.1, (1,2,3), (8)						
D. No.	C	N	C × N	C × C	D	Ds
1	1,2	3	13, 23	12	1	1,1,1,1,1,1

Design 3.2, (1,3,6), (8)						
D. No.	C	N	C × N	C × C	D	Ds
1	1,3	6	16,36	13	1	1,1,1,1,1,1

The other designs can be obtained by renaming the control and noise factors.

(b) $r = 1, s = 2$

Design 3.1, (1,2,3), (8)						
D. No.	C	N	C × N	C × C	D	Ds
1	1	2,3	12,13	-	1	1,1,1,1,1

The other designs can be obtained by renaming the control and noise factors.

Design 3.2, (1,3,6), (8)						
D. No.	C	N	C × N	C × C	D	Ds
1	1	3,6	13,16	-	1	1,1,1,1,1

For $p = 4$

(a) $r = 3, s = 1$

Design 4.3, (1,5,6,7), (12)						
D. No.	C	N	C × N	C × C	D	Ds
1	1,5,6	7	17,57,67	15,16,56	0.86	.73,.73,.73,.73,.67,.67,.67,.67,.67,.67

The other designs can be obtained by renaming the control and noise factors.

(b) $r = 1, s = 3$

Design 4.3, (1,5,6,7), (12)						
D. No.	C	N	C × N	C × C	D	Ds
1	1	5,6,7	15,16,17	-	0.95	.91,.91,.91,.91,.89,.89,.89

The other designs can be obtained by renaming the control and noise factors.

(c) $r = 2, s = 2$

Design 4.3, (1,5,6,7), (12)						
D. No.	C	N	C × N	C × C	D	Ds
1	1,5	6,7	16,17,56,57	15	0.88	.78,.78,.78,.78,.67,.67,.67,.67,.89

The other designs can be obtained by renaming the control and noise factors.

For $p = 5$

(a) $r = 4, s = 1$

Design 5.3, (1,2,3,5,6), (18)						
D. No.	C	N	C × N	C × C	D	Ds
1	1,2,3,6	5	15,25,35,65	12,13,16,23, 26,36	0.76	.44,.77,.44,.21,.45,.77,.44, .77,.45,.45, .21,.44,.45,.77,.44

Design 5.5, (1,2,5,6,7), (18)						
D. No.	C	N	C × N	C × C	D	Ds
1	1,5,6,7	2	12,52,62,72	15,16,17,56, 57,67	0.76	.77,.77,.45,.45,.44,.45,.45,.77, .77,.21, .44,.44,.44,.44,.21

(b) $r = 1, s = 4$

Design 5.9, (1,2,3,6,9), (14)						
D. No.	C	N	C × N	C × C	D	Ds
1	3	1,2,6,9	31,32,36,39	-	0.86	.91,.61,.79,.61,.76,.61, .76,.61,.79
2	9	1,2,3,6	91,92,93,96	-	0.86	.91,.61,.61,.76,.79,.61, .61,.79,.76

(c) $r = 3, s = 2$

Design 5.8 (1,3,5,6,8), (16)						
D. No.	C	N	C × N	C × C	D	Ds
1	1,3,5	6,8	16,18,36,38,56,58	13,15,35	0.68	.32,.17,.39,.32,.39,.39,.29,.39, .32,.29,.39,.39, .50,.32
2	1,3,8	5,6	15,16,35,36,85,86	13,18,38	0.68	.39,.17,.32,.32,.39,.29,.39,.39, .32,.39,.29,.32, .50,.39
3	3,5,6	1,8	31,38,51,58,61,68	35,36,56	0.68	.17,.32,.39,.39,.32,.32,.39,.29, .39,.39,.29,.39, .32,.50

(d) $r = 2, s = 3$

Design 5.9, (1,2,3,6,9), (14)						
D. No.	C	N	C × N	C × C	D	Ds
1	1,2	3,6,9	13,16,19,23,26,29	12	0.72	.46,.46,.43,.29,.43,.30,.43,.46, .57,.61,.33,.43
2	1,6	2,3,9	12,13,19,62,63,69	16	0.72	.46,.46,.29,.43,.43,.43,.46,.30, .61,.33,.57,.43