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The Direct Method for the Optimal Solution of a Transportation Problem

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Abstract

In this paper we discuss a new approach for solving both balanced and unbalanced transportation problem. The algorithm for proposed method discussed in this paper gives an initial as well as either optimal solution or near to optimal solution. Some numerical examples have been given to show the efficiency of the proposed method. Then results of the new approach are compared with the MODI method and we found that the proposed method gives either minimum or same optimal cost as compared to MODI's method and that too, in less iteration.

Key words: Balanced and unbalanced Transportation Problem; Basic feasible solution; Optimal solution; MODI method.

1. Introduction

Transportation Problem (TP) is one of the subclasses of Linear Programming Problems in which the objective is to transport various quantities of a single homogeneous commodity that are initially stored at various origins to different destinations in such a way that the total transportation cost is minimum. To achieve this objective, we must know the amount and location of available supplies and the quantities demanded. Also, we know the unit transportation cost of the commodity to be transported from various origins to destinations.

It was first studied by Hitchcock (1941) and then separately by Koopmans (1947) and finally placed in the framework of linear programming and solved by simplex method by Dantzig (1951). Since then, improved methods of solutions have been developed and the range of application has been steadily widened. It is now accepted as one of the important analytical and planning tools in business and industry. Several sorts of methods have been established for finding the optimal solution. Among them, some methods directly attain the optimal solution namely Zero Suffix Method, ASM-Method, *etc*. Also, it can be said that these methods obtain an optimal solution without disturbing degeneracy condition. They also require least iterations to reach optimality compared to the existing methods available in the literature. The degeneracy problem is also avoided by these methods. Recently, Pandian and Sudhakar proposed two different methods in 2010 and 2012 respectively for finding an optimal solution directly. However, the study on alternate optimal solutions is clearly limited in the literature of transportation except for Sudhakar, Arunnsankar and Karpagam (2012)

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who suggested a new approach for finding an optimal solution for transportation problems. Here we are proposing an easier approach for finding an optimal solution directly of the transportation problem as compared to MODI's method. Also, the proposed method is having lesser number of iterations and having easy arithmetical calculations.

2. Methodology

It is a simple and efficient method to obtain an optimal solution of transportation problem directly. The steps of the method are given below:

Step 1: Construct the transportation matrix from given transportation problem.

Step 2: Determine the smallest cost in the cost matrix of the transportation table. Let it be c_{ij}.

Step 3: Subtract the selected least cost c_{ij} from all the remaining cost in the matrix.

Step 4: Compare the minimum of supply or demand whichever is minimal then allocate the minimum supply or demand at the place of minimum value of related row or column. Let the minimum of supply (or demand) corresponds to i^{th} row (or j^{th} column). Let c_{ij} be the smallest cost in the i^{th} row (or j^{th} column). Allocate x_{ij} = min (a_i, b_j) in the $(i, j)^{th}$ cell. If tie occurs at the place of minimum value in supply or demand, then allocate at the maximum of supply or demand is observed.

Step 5: After performing Step 4, delete the i^{th} row (or j^{th} column) for further allocation where supply from a given source is depleted (or the demand for a given destination is satisfied).

Step 6: Repeat Step 4 and Step 5 for the reduced transportation table until all the demands are satisfied, and all the supplies are exhausted.

3. Numerical Problem

Problem 1: Consider the following cost minimizing transportation problem (balanced case):

	D1	**D2**	**D3**	**D4**	**Supply**
S1	13	18	30	8	**8**
S2	55	20	25	40	**10**
S3	30	6	50	10	**11**
Demand	**4**	**7**	**6**	**12**	**Total = 29**

	D1	**D2**	**D3**	**D4**	**Supply**
S1	7	12	24	2	**8**
S2	49	14	19	34	**10**
S3	24	0	44	4	**11**
Demand	**4**	**7**	**6**	**12**	**Total = 29**

Following allocations are obtained by applying the proposed method:

	D1	**D2**	**D3**	**D4**	**Supply**
S1	4			4	**8**
S2		4	6		**10**
S3		3		8	**11**
Demand	**4**	**7**	**6**	**12**	**Total = 29**

The Total cost from these allocations is 412 units.

Problem 2: Consider the following cost minimizing transportation problem (balanced case):

	D1	D2	D3	Supply
S1	11	9	6	**40**
S2	12	14	11	**50**
S3	10	8	10	**40**
Demand	**55**	**45**	**30**	**Total = 130**

	D1	D2	D3	Supply
S1	5	3	0	**40**
S2	6	8	5	**50**
S3	4	2	4	**40**
Demand	**55**	**45**	**30**	**Total = 130**

Following allocations are obtained by applying the proposed method:

	D1	D2	D3	Supply
S1		**10**	**30**	**40**
S2	**50**			**50**
S3	**5**	**35**		**40**
Demand	**55**	**45**	**30**	**Total = 130**

The Total cost from these allocations is 1200 units.

Problem 3: Consider the following cost minimizing transportation problem (unbalanced case):

Warehouse→

Plants	W1	W2	W3	Supply
A	28	17	26	**500**
B	19	12	16	**300**
Demand	**250**	**250**	**500**	

Warehouse→

Plants	W1	W2	W3	Supply
A	28	17	26	**500**
B	19	12	16	**300**
C	0	0	0	**200**
Demand	**250**	**250**	**500**	**Total=1000**

Following allocations are obtained by applying the proposed method:

Warehouse→

Plants	W1	W2	W3	Supply
A	50	250	200	**500**
B			300	**300**
C	200			**200**
Demand	**250**	**250**	**500**	**Total = 1000**

The Total cost from these allocations is 15650 units.

Problem 4: Consider the following cost minimizing transportation problem (Degeneracy case):

	D1	**D2**	**D3**	**D4**	**Supply**
S1	3	7	6	4	**5**
S2	2	4	3	2	**2**
S3	4	3	8	5	**3**
Demand	**3**	**3**	**2**	**2**	**Total = 10**

	D1	**D2**	**D3**	**D4**	**Supply**
S1	1	5	4	2	**5**
S2	0	2	1	0	**2**
S3	2	1	6	3	**3**
Demand	**3**	**3**	**2**	**2**	**Total = 10**

Following allocations are obtained by applying the proposed method:

	D1	**D2**	**D3**	**D4**	**Supply**
S1	3			2	**5**
S2			2	ε_1	**2**
S3	ε_2	3			**3**
Demand	**3**	**3**	**2**	**2**	**Total=10**

Total transportation cost = $(3 \times 3) + (2 \times 4) + (2 \times 3) + (\varepsilon_1 \times 2) + (\varepsilon_2 \times 4) + (3 \times 3)$
$= 32 + 2\varepsilon_1 + 4\varepsilon_2$
$= 32$ as $\varepsilon_1 \to 0$ and $\varepsilon_2 \to 0$

The total transportation cost is 32 units.

4. Results and Comparison

Comparison of total cost of Transportation Problem of above examples between MODI method and proposed method is:

Problem#	**Type of Problem**	**Problem Dimension**	**MODI's Method**	**Proposed Method**
1	Balanced	3×4	**412**	**412**
2	Balanced	3×3	**1320**	**1200**
3	Unbalanced	2×3	**15700**	**15650**
4	Degeneracy	4×3	**32**	**32**

5. Conclusion

In this paper, a simple and more efficient method is determined to solve both the balanced and unbalanced transportation problem. Also, the proposed method gives an optimal transportation cost directly without solving the Initial Basic Feasible Solution. The new approach finds an optimal cost of the transportation problem in a very short time-period and having lesser computations as compared to MODI method. Also, the problem of degeneracy can be handled by this proposed method. Thus, our study clearly shows that the new approach is more efficient and reliable for getting an optimal solution of various types of transportation problems as compared to the well-known existing methods present in the literature.

Finally, the proposed method presented in this paper claims its wide application in solving transportation problems of higher order matrices.

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