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Probability Distributions in Image Segmentation

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Abstract

In this paper we review some image segmentation methods based on mixture distributions. Here, it is considered that pixel intensities in each image region follow a probability distribution. The distribution may be platykurtic or mesokurtic or leptykurtic. The whole image is characterized by a probabilistic mixture model. The number of components (image regions) in each image is obtained through K-means/hierarchal clustering algorithm. The model parameters are estimated by deriving the updated equations of the EM algorithm. The segmentation of the image is done by maximizing the component likelihood. The performance of the different algorithms is studied by computing the segmentation performance metrics like, PRI, VOI, and GCE for five images randomly selected from Barkley image data set. The experimental results and comparative study show that these methods outperform the non-parametric methods.

Keywords: Image segmentation, Mixture distributions, E.M. algorithm, Segmentation performance metrics.

1. Introduction

The optical appearance of something produced in a mirror or through a lens is known as image. The concept of digital image was found in literature as early as in 1920.In low level image analysis, the entire image is considered as a union of several image regions .In each image region the image data is quantized by pixel intensities. The pixel intensity $z = f(x, y)$ for a given point (pixel), z is a random variable, because of the fact that the brightness measured at a point in the image is influenced by various random factors like vision, lighting, moisture, environmental conditions etc. Digital image is a matrix, where each number represents the brightness at regularly spaced points in the image. These points are called pixels and the brightness value of a pixel is called its grey level.

The aim of image processing applications is to extract important features from image data from which a description, interpretation or understanding of the scene can be provided by the machine. Image analysis helps to find the relationship between the objects inside the image. The first step of image analysis is to divide the image into regions so that various features such as size, shape, color, texture can be measured, and these features in turn can be used as inputs for classification. Image analysis involves: (i) feature extraction, and (ii) segmentation. Image segmentation refers to decomposition of a scene into different components. Segmentation is a process of partitioning the image into non-intersecting regions such that each region is homogenous. Several Segmentation techniques have been developed and utilized for image analysis, but there is no unique segmentation procedure, which serve

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all the situations. The uses of image segmentation are Image Understanding (content identification) and Image retrieval. Image Segmentation is extensively used in medical diagnostics, remote sensing, robotics, filming and video, industrial automation and animation (Jan Puzicha ,1999, Martin, 2001).

The image segmentation methods can be grouped into three categories, viz., (1) Histogram, Threshold and edge based methods, (2) Model based image segmentation methods, and (3) Image segmentation based on other methods (graph, neural networks, fuzzy logic, genetic algorithms, saddle points etc.). These methods can further be classified into two categories *viz*., parametric and non-parametric image segmentation methods. Parametric (model based) image segmentation methods are more efficient compared to the non-parametric methods of segmentation. In model-based image segmentation, whole image is characterized by a finite mixture of probability distributions by ascribing a probability distribution to the pixel intensities of each image region (Figueiredo et al., 1999, Jacob, Goldberg, 2002, Lei et al., 2003, Pal and Pal, 1993) have mentioned that there is no unique image segmentation method which can serve all images. This is true, since the pixel intensities in each image region follow mesokurtic, leptykutric or platykurtic, symmetric or asymmetric distributions (Abhir Bhalerao and Roland Wilson, 2003).

In this article we discuss different image segmentation methods based on mixtures of different probability distributions which are used for analyzing several images. We also discuss the updated equations of the EM-algorithms associated with the finite mixture models. The performance of some image segmentation methods are also discussed (Unnikrishnan, et al., 2007). In conclusions a comparative study of different image segmentation methods is presented.

2. Image Segmentation Algorithm

For segmenting the image into image regions, we adopt the following steps after ascribing the suitable probability model to the feature of each image region.

Step 0) Identify a suitable probability model for the feature (pixel intensity) using criteria given for Pearsonian system of equations (Johnson et al., 1992). In this system consider a quadratic equation of the form

$$b_0 + b_1 X + b_2 X^2 = 0 \qquad \text{where,} \qquad b_0 = -\frac{\mu_2(4\mu_2\mu_4 + 3\mu_3^2)}{2(5\mu_2\mu_4 - 9\mu_2^3 - 6\mu_3^6)},$$

$$b_1 = -\frac{2(\mu_2\mu_4 - 3\mu_3^2 + 6\mu_2^3)}{2(5\mu_2\mu_4 - 9\mu_2^3 - 6\mu_3^2)}, \qquad b_2 = -\frac{2(\mu_2\mu_4 - 3\mu_3^2 + 6\mu_2^3)}{2(5\mu_2\mu_4 - 9\mu_2^3 - 6\mu_3^6)}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}, \ \beta_2 = -\frac{\mu_2^4}{\mu_3^2}, \text{and}\ K = \frac{b_1^2}{(4b_0b_2)}$$

If $K \to \pm\infty$ then Type III ; If $K = -\infty$ then Type I; If $K =$ 0 to 1 then Type IV; If $K = 1$ to ∞ then Type VI; If $K = 0$ & $\beta_2 < 3$ then Type II; If $K = 0$ & $\beta_2 = 3$ Normal curve; If $K = 0$ & $\beta_2 > 3$ then Type VII.

Step 1) Plot the histogram of the whole image.

Step 2) Obtain the initial estimates of the model parameters using clustering algorithm and moment estimates for each image region.

Step 3) Obtain the refined estimates of the model parameters μ_i, σ_i^2 and α_i for i =1, 2, … , K, by using the EM algorithm.

Step 4) Assign each pixel into the corresponding j^{th} region (segment) according to the Maximum likelihood of the j^{th} component L_j where, L_j is component likelihood function of j^{th} region.

3. Image Segmentation Method Based On Gaussian Mixture Model

Much emphasis is given for image analysis through finite Gaussian mixture model. In finite Gaussian mixture model each image region is characterized by a Gaussian distribution and the entire image is considered to be a mixture of these Gaussian components. Several researchers assumed that the whole image is characterized by Gaussian mixture model in which the pixel intensities of each image region follow a Gaussian distribution. Image segmentation methods based on Gaussian or Gaussian Mixture models were studied by Yamazaki. (1998), Jan Puzicha. (1999), Figureido et al. (1999), Farnoosh et al. (2008), Jacob et al. (2002), Permuter et al. (2003), Yudi Augusta (2003), Blekas, et al. (2005), and others.

The probability density function of the pixel intensity of the image region which follows a Gaussian distribution is given by,

$$f(z;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}$$

$$-\infty<z<\infty;\ -\infty<\mu<\infty;\ 0<\sigma \tag{1}$$

The mean of the pixel intensities in the image region is μ and variance of the pixel intensities in the image region is σ^2 .Since the whole image is a collection of K image regions the pixel intensities in the whole image follows a Gaussian mixture model with Probability density function

$$g(z) = \sum_{i=1}^{K} \alpha_i f_i(z\ ;\mu_i,\sigma_i);\quad -\infty < z < \infty\ . \tag{2}$$

where, α_i is the component weight in the mixture model such that $\sum_{i=1}^{K}\alpha_i = 1$ and $f_i(\underline{z}\ ;\mu_i,\ \sigma_i)$is as given in equation (1).

The model parameters are estimated by using the EM algorithm (Mclanchlan and Krishnan, 1997). The updated estimates of the model parameters are:

$$\alpha_i^{(l+1)} = \frac{1}{\mathrm{K}} \sum_{i=1}^{K} p(z_i;\theta^{(l)})\ ,$$

$$\mu_i^{(l+1)} = \frac{\sum_{i=1}^{K} z_i\, p(z_i;\theta^{(l)})}{\sum_{i=1}^{K} p(z_i;\theta^{(l)})}\ ,$$

and $$(\sigma_i^2)^{(l+1)} = \frac{\sum_{i=1}^{K} p(z_i;\theta^{(l)})(z_i-\mu_i^{(l+1)})^2}{\sum_{i=1}^{K} p(z_i;\theta^{(l)})}$$

where, $p(z_i / \theta^{(l)}) = \dfrac{\alpha_i f_i(z;\mu_i,\sigma_i)}{\sum_{i=1}^{K} \alpha_i f_i(z;\mu_i,\sigma_i)}$

Usually the initialization of the parameters is done by segmenting the whole image into *K* image regions with *K*-means / hierarchal clustering and using the following initial estimates for each image regions as

$\alpha_i =$ 1/*K*, for all *i*= 1,2….*K*,

$\mu_i =$ Sample mean of the i^{th} region,

$\sigma_i^2 =$ Sample variance of the i^{th} region.

4. Image Segmentation Method Based on New Symmetric Mixture Distribution

Seshashayee et al. (2011a), Srinivasa Rao et al. (2012) have used mixture of new symmetric distribution for image segmentation. To model the pixel intensities of the image region which are distributed as platykurtic symmetric distribution, it is assumed that the pixel intensities of the region follow a new symmetric distribution given by Srinivasa Rao, K, et al. (1997).The probability density function of the pixel intensity is given by

$$f(z,\mu,\sigma^2) = \frac{\left(2+\left(\frac{z-\mu}{\sigma}\right)^2\right) e^{\frac{-1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}}{3\sigma\sqrt{2\pi}} \tag{3}$$

This distribution is symmetric aboutμ and the distribution function is

$$F(z) = \frac{2}{3}\Phi\left(\frac{z-\mu}{\sigma}\right) - \frac{1}{3} e^{\frac{-1}{2}\left(\frac{z-\mu}{\sigma}\right)^2} \left[1 + \frac{(z-\mu)}{\sigma\sqrt{2\pi}}\right]$$

where, $\Phi\left(\frac{z-\mu}{\sigma}\right)$ is the distribution function of the standard normal variate.

The kurtosis of the distribution is $\beta_2 = 2.52$ (4)

The entire image is a collection of regions which are characterized by new symmetric distribution. Here, it is assumed that the pixel intensities of the whole image follows *K* component mixture of new symmetric distribution and its probability density function is of the form

$$p(z) = \sum_{i=1}^{K} \alpha_i f_i(z / \mu_i, \sigma_i^2) \tag{5}$$

where, *K* is number of regions, $0 \le \alpha_i \le 1$ are weights such that$\sum\alpha_i = 1$ and $f_i(z,\mu,\sigma^2)$ is as given in equation (4). α_i is the weight associated with i^{th} region in the whole image.

Estimation of the Model Parameters by EM Algorithm:

The updated equations of the parameters in each image region are obtained for the EM algorithm as follows.

The updated equation of α for $(l+1)^{\text{th}}$ iteration is

$$\alpha_i^{(l+1)} = \frac{1}{N}\sum_{s=1}^{N} t_i(z_s,\theta^{(l)}) = \frac{1}{N}\sum_{s=1}^{N}\left[\frac{\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}\right] \tag{6}$$

The updated equation of μ_i at $(l+1)^{\text{th}}$ iteration is

$$\mu_i^{(l+1)} = \frac{\sum_{s=1}^{N} z_s t_i(z_s,\theta^{(l)}) - \sum_{s=1}^{N} t_i(z_s,\theta^{(l)})\left(\frac{2(\sigma_i^2)^{(l)}\left(z_s-\mu_i^{(l)}\right)}{2(\sigma_i^2)^{(l)}+\left(z_s-\mu_i^{(l)}\right)^2}\right)}{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})} \tag{7}$$

where, $t_i(z_s,\theta^{(l)}) = \dfrac{\alpha_i^{(l+1)} f_i(z_s,\mu_i^{(l)},\left(\sigma_i^2\right)^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l+1)} f_i(z_s,\mu_i^{(l)},\left(\sigma_i^2\right)^{(l)})}$

The updated equation of σ_i^2 at $(l+1)^{\text{th}}$ iteration is

$$\left(\sigma_i^2\right)^{(i+1)} = \frac{2\sum_{s=1}^{N}(z_s-\mu_i^{(i+1)})^2\left(\frac{1}{2}-\frac{(\sigma_i^2)^{(i)}}{\left(2(\sigma_i^2)^{(i)}+\left(z_s-\mu_i^{(i+1)}\right)^2\right)^2}\right)(t_i(z_s,\theta^{(i)}))}{\sum_{s=1}^{N} t_i(z_s,\theta^{(i)})} \tag{8}$$

5. Image Segmentation Method Based on Generalized New Symmetric Mixture Distribution

Seshashayee, et al. (2011b) have used generalized new symmetric distribution and its mixture distribution in modeling the pixel intensities of an image. To model the pixel intensities of the image region, it is assumed that the pixel intensities of the region follow a generalized new symmetric distribution.

The probability of the density function of the pixel intensities in the whole image is

$$p(z) = \sum_{i=1}^{K}\alpha_i f_i(z/\mu_i,\sigma_i^2,r_i) \tag{9}$$

where, K is number of regions, $0\le \alpha_i \le 1$ are weights such that $\sum\alpha_i = 1$. α_i is the weight associated with i^{th} region of the image, $f_i(z,\mu_i,\sigma_i^2,r_i)$ is the probability density function of the pixel intensities of the i^{th} image region, which is characterized by a generalized new symmetric distribution. The probability density function of the pixel intensity is

$$f(z,\mu,\sigma^2,r) = \frac{\left(2r+\left(\frac{z-\mu}{\sigma}\right)^2\right)^r e^{\frac{-1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}}{\sigma(2r)^r(2\Pi)^{\frac{1}{2}}+\sum_{j=1}^{r}\binom{r}{j}(2r)^{r-j}2^{j+\frac{1}{2}}\Gamma\left(j+\frac{1}{2}\right)\sigma} \tag{10}$$

$$-\infty < z < \infty, -\infty < \mu < \infty, \sigma > 0,\ \text{r} = 0,1,2...$$

For different values of the parameters the various shapes of probability curves associated with the generalized new symmetric distribution are shown in Figure 1.

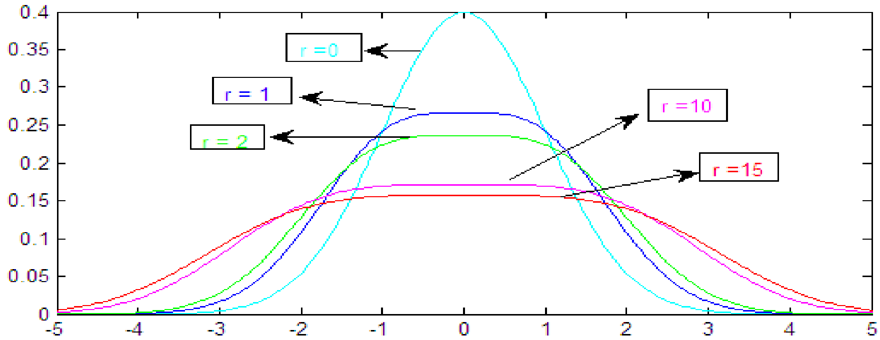

Figure 1: Probability curves of new symmetric distribution

Estimation of the Model Parameters using EM Algorithm

In this section, the estimates of the model parameters through EM algorithm are obtained. Here, it is assumed that the pixel intensities of the whole image follow a generalized new symmetric mixture distribution. The updated equations of the parameters in each image region are obtained for the EM algorithm as follows

The updated equation of α for $(l+1)^{th}$ iteration is

$$\alpha_i^{(l+1)} = \frac{1}{N}\sum_{s=1}^{N} t_i(z_s,\theta^{(l)}) = \frac{1}{N}\sum_{s=1}^{N}\left[\frac{\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}\right] \tag{11}$$

The updated equation of μ_i at $(l+1)^{th}$ iteration is

$$\mu_i^{(l+1)} = \frac{\sum_{s=1}^{N} z_s t_i(z_s,\theta^{(l)}) - \sum_{s=1}^{N} t_i(z_s,\theta^{(l)})\left(\frac{2r_i(\sigma_i^2)^{(l)}\left(z_s-\mu_i^{(l)}\right)}{2r_i(\sigma_i^2)^{(l)}+\left(z_s-\mu_i^{(l)}\right)^2}\right)}{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})} \tag{12}$$

where, $t_i(z_s,\theta^{(l)}) = \dfrac{\alpha_i^{(l+1)} f_i(z_s,\mu_i^{(l)},\left(\sigma_i^2\right)^{(l)},r_i)}{\sum_{i=1}^{K}\alpha_i^{(l+1)} f_i(z_s,\mu_i^{(l)},\left(\sigma_i^2\right)^{(l)},r_i)}$

The updated equation of σ_i^2 at $(l+1)^{th}$ iteration is

$$\left(\sigma_i^2\right)^{(l+1)} = \frac{2\sum_{s=1}^{N}(z_s-\mu_i^{(l+1)})^2\left(\frac{1}{2}-\frac{r_i\left(\sigma_i^2\right)^{(l)}}{\left(2r_i(\sigma_i^2)^{(l)}+\left(z_s-\mu_i^{(l+1)}\right)^2\right)^2}\right)\left(t_i(z_s,\theta^{(l)})\right)}{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})} \tag{13}$$

where, $t_i(z_s,\theta^{(l)}) = \dfrac{\alpha_i^{(l+1)} f_i(z_s,\mu_i^{(l+1)},\left(\sigma_i^2\right)^{(l)},r_i)}{\sum_{i=1}^{K}\alpha_i^{(l+1)} f_i(z_s,\mu_i^{(l+1)},\left(\sigma_i^2\right)^{(l)},r_i)}$

6. Image Segmentation Based on Generalized Laplace Mixture Model

Jyothirmayi, et al. (2015) have used generalized Laplace mixture distribution for image segmentation. The pixel intensities are assumed to follow a generalized Laplace distribution given by Srinivasa, Rao et al. (1997).

The probability density function of the pixel intensity is given by

$$f(x,\mu,\sigma^2) = \left(\frac{\left(r^2+\dfrac{(x-\mu)^2}{\sigma^2}\right)^r e^{-\left|\frac{x-\mu}{\sigma}\right|}}{2\sigma\sum_{k=0}^{r}\binom{r}{k} r^{2(r-k)}(2k!)}\right) \tag{14}$$

where, $-\infty<x<\infty$, $-\infty<\mu<\infty$, $\sigma>.0$

The probability density function of pixel intensities in whole image is

$$p(x) = \sum_{i=1}^{k}\alpha_i f_i(x_s,\mu_i,\sigma_i^2) \tag{15}$$

where, k is the number of regions, α_i is the weight which lie in the range of (0,1) such that sum of α_i in all clusters is equal to 1.

Estimation of Parameters through EM Algorithm

The updated equations of the parameters in each image region are obtained for the EM algorithms are:

The updated equation of α_i for $(l+1)^{\text{th}}$ iteration is

$$\alpha_i^{(l+1)} = \frac{1}{N}\sum_{s=1}^{N} T_i(x_s,\theta^{(l)}) = \frac{1}{N}\sum_{s=1}^{N}\left[\frac{\alpha_i^{(l)} f_i(x_s,\theta^{(l)})}{\sum_{i=1}^{k}\alpha_i^{(l)} f_i(x_s,\theta^{(l)})}\right] \tag{16}$$

The updated equation of μ_i at $(l+1)^{\text{th}}$ iteration is

$$\sum_{s=1}^{N} T_i(x_s,\theta^l)\frac{x_s-\mu_i}{\sigma_i\,|x_z-\mu_i|} - \sum_{s=1}^{N} T_i(x_s,\theta^l)\frac{[2(x_s-\mu_i)]}{r^2\sigma_i^2+(x_s-\mu_i)^2} = 0 \tag{17}$$

where, $T_i(x_s,\theta^l) = \dfrac{\alpha_i^{l+1} f_i(x_s,\theta^l)}{\sum_{i=1}^{k}\alpha_i^{l+1} f_i(x_s,\theta^l)}$

For updating σ_i^2 $Q(\theta;\theta^l)$ is differentiated with respect to σ_i^2 and equated to 0

$$\frac{\partial}{\partial\sigma_i^2} Q(\theta;\theta^l) = 0$$

This implies

$$\sum_{s=1}^{N}\left[\frac{(x_s-\mu_i)^2}{(r^2\sigma_i^2+(x_s-\mu_i)^2\sigma_i^2}-\left|\frac{x_s-\mu_i}{2\sigma_i^2}\right|\right]T_i(x_s,\theta^{(l)})=0 \tag{18}$$

where, $T_i(x_s,\theta^{(l)})=\dfrac{\alpha_i^{(l+1)}f_i(x_s,\theta^{(l)})}{\sum_{i=1}^{k}\alpha_i^{(l+1)}f_i(x_s,\theta^{(l)})}$

Jyothirmayi, et al. (2016, 2017) have developed and analyzed image segmentation methods based on doubly truncated generalized Laplace mixture distribution.

7. Image Segmentation Based on Finite Generalized Gaussian Mixture Model

Prasad Reddy et al.(2007), Srinivas Yerramalle, et al. (2010a, 2010b)have used Generalized Gaussian mixture model for image segmentation. Here, it is assumed that the pixel intensities inside each image region are characterized by Generalized Gaussian distribution and the entire image is characterized by finite Generalized Gaussian Mixture distribution. The probability density function of the Generalized Gaussian distribution is

$$f(z\mid\mu,\sigma,P)=\frac{1}{2\Gamma(1+\frac{1}{P})A(P,\sigma)}e^{-\left|\frac{(Z_i-\mu_i)}{A(P,\sigma)}\right|^P} \tag{19}$$

where, $\sigma>0,\ A(P,\sigma)=\left[\dfrac{\sigma^2\Gamma(1/P)}{\Gamma(3/P)}\right]^{\frac{1}{2}}\qquad -\infty<z<\infty,-\infty<\mu<\infty,\ P>0$ (20)

The parameter μ is the mean, the function $A(P,\sigma)$ is an scaling factor which allows that the Variance of Z = σ^2, and ‘P’ is the shape parameter.

Estimation of the Model Parameters through EM Algorithm

The parameters $\mu_{i,}\sigma_{i,}\alpha_i$ for i =1,2,…,K are obtained by using the EM algorithm .The updating equations of the parameters in each image region are obtained as follows

$$\alpha_i^{(l+1)}=\frac{1}{N}\sum_{s=1}^{N}\left(\frac{\alpha_i^{(l)}f_i(z_s,\theta^{(l)})}{\sum_{i=1}^{K}f_i(z\ ,\theta^{(l)})}\right) \tag{21}$$

$$\mu_k^{(l+1)}=\frac{\sum_{s=1}^{N}z_s\alpha_k^{(l)}f_k(z_z,\theta^{(i)})^{\frac{1}{p_k-1}}}{\sum_{s=1}^{N}\alpha_k^{(l)}f_k(z_s,\theta^{(i)})^{\frac{1}{p_k-1}}} \tag{22}$$

$$\sigma_i^{(l+1)} = \left[\frac{\sum_{s=1}^{N} t^l_i (z_s, \theta^{(l)}) \left(\frac{\Gamma(3/P_i)}{P\Gamma(1/P_i)} \right) \left| z_s - \mu_i^{(l)} \right|^{\frac{1}{P_i}}}{\sum_{s=1}^{N} t_i (z_s, \theta^{(l)})} \right]^{\frac{1}{P_i}} \tag{23}$$

where, $t_i(z_s;\theta^{(l)}) = = \frac{\alpha_i^{(l)} f_i(z_s,\theta^{(l)})}{\sum_{i=1}^{K} \alpha_i^{(l)} f_i(z_s,\theta^{(l)})}$ (24)

8. Image Segmentation Based on Finite Doubly Truncated Gaussian Mixture Model

Srinivas.Yerramalle and Srinivasa Rao. K. (2007a, 2007b) have considered an image segmentation algorithm by assuming that the pixel intensities of the entire image follow a finite doubly truncated Gaussian Mixture distribution. The doubly Truncated Gaussian distribution with probability density function is

$$g_i(z,\theta_i) = \frac{\frac{1}{\sqrt{2\pi}\sigma_i} e^{\frac{-1}{2}\left(\frac{(z_i-\mu_i)}{\sigma_i}\right)^2}}{B-A}; \quad z_L < z < z_M$$

$$B = \int_{-\infty}^{z_M} \frac{e^{\frac{-1}{2}\left(\frac{(z_i-\mu_i)}{\sigma_i}\right)^2}}{\sqrt{2\pi}\,\sigma_i} dz_i \quad ; \quad A = \int_{-\infty}^{z_L} \frac{e^{\frac{-1}{2}\left(\frac{(z_i-\mu_i)}{\sigma_i}\right)^2}}{\sqrt{2\pi}\,\sigma_i} dz_i \tag{25}$$

As a result of this, the pixel intensities in the entire image follow a finite doubly Truncated Gaussian distribution with Probability density function of the form

$$h(z,\theta) = \sum_{i=1}^{K} \alpha_i g_i(z,\theta_i) \tag{26}$$

where, $g_i(z,\theta_i)$ is as given in equation (25) and $0<\alpha_i<1$, $\sum_{i=1}^{K} \alpha_i = 1$.

Estimation of Model Parameters by EM Algorithm

The updating equations of the parameters in each image region are as follows:
For updating α_k, we have,

$$\alpha_k^{(l+1)} = \frac{1}{N} \sum_{s=1}^{N} \frac{\alpha_k^{(l)}}{H(z_M,\theta^{(l)}) - H(z_L,\theta^{(l)})} \tag{27}$$

where $H(z_M,\theta^{(l)}) = \int_{-\infty}^{z_M} \alpha_i g_i(z_s,\theta^{(l)}) dz$

$$H(z_L,\theta^{(l)}) = \int_{-\infty}^{z_L} \alpha_i g_i(z_s,\theta^{(l)}) dz$$

For updating $\mu_i^{(l)}$ we have,

$$\mu_k^{(l+1)} = \left[\mu_k^{(l)} + 2\sigma_k^{2(l)} \left(\frac{f(z_M) - f(z_L)}{B-A} \right) \right]$$

where, $A = \int_{-\infty}^{z_L} \frac{e^{\frac{-1}{2}\left(\frac{(z-\mu)}{\sigma}\right)^2}}{\sqrt{2\pi}\,\sigma} dz$; $B = \int_{-\infty}^{z_M} \frac{e^{\frac{-1}{2}\left(\frac{(z-\mu)}{\sigma}\right)^2}}{\sqrt{2\pi}\,\sigma} dz$ (28)

For updating σ_k^2 , we have,

$$\sigma_k^{2(l+1)} = \mu_i^{2(l)} + (1-\mu_i^{(l)}) \frac{\pi_i^{(l)}}{D} \left(\frac{\sigma_i^{2(l)}}{F(z_M, \theta^{(l)}) - F(z_L, \theta^{(l)})} \right)$$
$$-\sigma_i^{2(l)} \frac{\pi_i^{(l)}}{D} \left(\frac{z_M f_i(z_M, \theta^{(l)}) - z_L f_i(z_L, \theta^{(l)})}{F(z_M, \theta^{(l)}) - F(z_L, \theta^{(l)})} \right)$$
$$-\frac{\pi_i^{(l)}}{D} \left(\frac{2\mu_k \mu_i^{(l+1)}}{F(z_M, \theta^{(l)}) - F(z_L, \theta^{(l)}} - \frac{2\mu_k \sigma_i^{2(l)}}{F(z_M, \theta^{(l)}) - F(z_L, \theta^{(l)}} \right) + \mu_i^{2(l+1)}$$

where, $D = \frac{1}{(B-A)} \Big((1+\mu_k) \big[(f_k(z_L, \theta^{(l)}) - f_k(z_M, \theta^{(l)} \big] + (z_M f_k(z_M, \theta^{(l)}) - z_L f_k(z_L, \theta^{(l)})) \Big)$

(29)

9. Image Segmentation Method Based on Pearsonian Type I Distribution

Chandra Sekhar et al. (2014) have considered that the pixel intensities of the image region follows a Pearson Type I distribution. The probability density function of the pixel intensity is

$$f(z, a_{i1}, a_{i2}, m_{i1}, m_{i2}) = \frac{a_{i1}^{m_{i1}} a_{i2}^{m_{i2}} \left(1 + \frac{z}{a_{i1}}\right)^{m_{i1}} \left(1 - \frac{z}{a_{i2}}\right)^{m_{i2}}}{(a_{i1} + a_{i2})^{(m_{i1}+m_{i2}+1)} B(m_{i1}+1, m_{i2}+1)} \tag{30}$$

$$-\infty < m_{i1} < \infty, -\infty < m_{i2} < \infty, a_{i1} \le z \le a_{i2}$$

The entire image is a collection of regions which are characterized by Pearson Type I distribution. Here, it is assumed that the pixel intensities of the whole image follows a *K*–component mixture of Pearson type I distribution and its probability density function is of the form

$$p(z) = \sum_{i=1}^{K} \alpha_i \, f_i(z, a_{i1}, a_{i2}, m_{i1}, m_{i2}) \tag{31}$$

where, K is number of regions,$0 \le \alpha_i \le 1$ are weights such that$\sum \alpha_i = 1$ and $f_i(z, a_{i1}, a_{i2}, m_{i1}, m_{i2})$ is as given in equation(30). α_i is the weight associated with i^{th} region in the whole image.

Estimation of the Model Parameters by EM Algorithm:

The updated equation of α_i for $(l+1)^{\text{th}}$ iteration is

$$\alpha_i^{(l+1)} = \frac{1}{N}\sum_{s=1}^{N} t_i(z_s,\theta^{(l)}) = \frac{1}{N}\sum_{s=1}^{N}\left[\frac{\alpha_i^{(l)} f_j(z_z,\theta^{(l)})}{\sum_{i=1}^{K} a_i^{(l)} f_i(z_s,\theta^{(l)})}\right] \tag{32}$$

The updated equation of m_{i1} at $(l+1)^{\text{th}}$ iteration is

$$m_{i1}^{(l+1)} = \frac{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})\left[m_{i1}^{(l)}\log\left(\frac{a_{i1}+z_s}{a_{i1}+a_{i2}}\right)-1\right]}{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})\left(\psi_0(m_{i1}^{(l)})-\psi_0(m_{i1}^{(l)}+m_{i2}^{(l)}+2)\right)} \tag{33}$$

where, $t_i(z_s,\theta^{(l)}) = \dfrac{\alpha_i^{(l+1)} f_i(z_s,\theta^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l+1)} f_i(z_s,\theta^{(l)})}$

The updated equation of m_{i2} at $(l+1)^{\text{th}}$ iteration is

$$m_{i2}^{(l+1)} = \frac{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})\left[m_{i2}^{(l)}\log\left(\frac{a_{i2}-z_s}{a_{i1}+a_{i2}}\right)-1\right]}{\sum_{s=1}^{N} t_i(z_s,\theta^{(l)})\left(\psi_0(m_{i2}^{(l)})-\psi_0(m_{i1}^{(l)}+m_{i2}^{(l)}+2)\right)} \tag{34}$$

where, $t_i(z_s,\theta^{(l)}) = \dfrac{\alpha_i^{(l+1)} f_i(z_s,\theta^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l+1)} f_i(z_s,\theta^{(l)})}$

10. Image Segmentation Method Based on Pearsonian Type III Distribution

To model the pixel intensities of the image region Chandra Sekhar, (2015) has assumed that the pixel intensities of the region follow a Pearson Type III distribution. The probability density function of the pixel intensity is

$$f_i(z/a_i,q_i) = \frac{(q_i a_i)^{q_i a_i+1)}}{e^{q_i a_i}\Gamma(q_i a_i+1)} e^{-q_i z_s}\left(1+\frac{z_s}{a_i}\right)^{-q_i a_i}, \qquad -a_i \le z_s < \alpha \tag{35}$$

where, Γ is a gamma function.

Here, it is assumed that the pixel intensities of the whole image follow a K–component mixture of Pearson type III distribution and its probability density function is of the form

$$p(z) = \sum_{i=1}^{K}\alpha_i\, f_i(z\,|\,a_i,q_i) \tag{36}$$

where, K is number of regions ,$0 \leq \alpha_i \leq 1$ are weights such that $\sum \alpha_i = 1$ and $f_i(z \mid a_i, q_i)$ is as given in equation (35). α_i is the weight associated with i^{th} region in the whole image.

Estimation of the Model Parameters by EM Algorithm

The updated equation of α_i for $(l+1)^{\text{th}}$ iteration is

$$\alpha_i^{(l+1)} = \frac{1}{N}\sum_{s=1}^{N} t_i(z_s, \theta^{(l)}) = \frac{1}{N}\sum_{s=1}^{N}\left[\frac{\alpha_i^{(l)} f_i(z_s, \theta^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l)} f_i(z_s, \theta^{(l)})}\right] \tag{37}$$

The updated equation of a_i for $(l+1)^{\text{th}}$ iteration is

$$a_i^{(l+1)} = \sum_{s=1}^{N} \frac{t_i(z_s, \theta^{(l)})}{\left[\frac{q_i^{(l)} z_s}{a_i^{(l)} + z_s} + q_i^{(l)}\Gamma\left(q_i^{(l)} a_i^{(l)} + 1\right) - q_i^{(l)} \log\left(q_i^{(l)}\left(a_i^{(l)} + z_s\right)\right)\right] t_i(z_s, \theta^{(l)})} \tag{38}$$

where $t_i(z_s, \theta^{(l)}) = \left[\frac{\alpha_i^{(l+1)} f_i(z_s, \theta^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l+1)} f_i(z_s, \theta^{(l)})}\right]$

The updated equation of q_i for $(l+1)^{\text{th}}$ iteration is

$$q_i^{(l+1)} = \frac{\sum_{s=1}^{N} q_i\left[a_i^{(l)}\Gamma\left(q_i^{(l)} a_i^{(l)} + 1\right) + (a_i^{(l)} + z_s) - a_i^{(l)} \log\left(q_i^{(l)} a_i^{(l)}\left(\frac{z_s + a_i^{(l)}}{a_i^{(l)}}\right)\right)\right] t_i(z_s, \theta^{(l)})}{a_i^{(l)}\sum_{s=1}^{N} t_i(z_s, \theta^{(l)})} - 1 \tag{39}$$

where $t_i(z_s, \theta^{(l)}) = \left[\frac{\alpha_i^{(l+1)} f_i(z_s, \theta^{(l)})}{\sum_{i=1}^{K}\alpha_i^{(l+1)} f_i(z_s, \theta^{(l)})}\right]$

11. Image Segmentation Method Based on Pearsonian Type VI Distribution

To model the pixel intensities of the animal and human image regions, Srinivasa Rao. K. et al, (2014) have assumed that the pixel intensities of the region follow a Pearson Type VI distribution (PTVID). The probability density function of the pixel intensity is

$$f_i(z / a_{i1}, q_{i1}, q_{i2}) = \frac{\left(z_s - a_i\right)^{q_{i2}}\left(z_s\right)^{-q_{i1}}}{(a_i)^{(q_{i2} - q_{i1} + 1)} B(q_{i1} - q_{i2} - 1, q_{i2} + 1)} \qquad a_i \leq z_i < \alpha \tag{40}$$

Here, it is assumed that the pixel intensities of the whole image follow a K – component mixture of Pearson Type VI distribution and its probability density function is of the form

$$p(z) = \sum_{i=1}^{K} \alpha_i \; f_i(z / a_{i1}, q_{i1}, q_{i2}) \tag{41}$$

where, K is number of regions, $0 \leq \alpha_i \leq 1$ are weights such that $\sum \alpha_i = 1$ and $f_i(z / a_{i1}, q_{i1}, q_{i2})$ is as given in equation (40). α_i is the weight associated with i^{th} region in the whole image.

Estimation of the Model Parameters by EM Algorithm:

The updated equation of α_i for $(l+1)^{\text{th}}$ iteration is

$$\alpha_i^{(l+1)} = \frac{1}{N} \sum_{s=1}^{N} \left(\frac{\alpha_i^{(l)} f_i(z_s, \theta^{(l)})}{\sum_{i=1}^{K} \alpha_i^{(l)} f_i(z_s, \theta^{(l)})} \right) \tag{42}$$

The updated equation of a_i at $(l+1)^{\text{th}}$ iteration is

$$a_i^{(l+1)} = \sum_{s=1}^{N} \left[\frac{-(q_{i2}^{(l)} - q_{i1}^{(l)} + 1)\left(z_s - a_i^{(l)}\right) t_i(z_s, \theta^{(l)})}{t_i(z_s, \theta^{(l)}) q_{i2}^{(l)}} \right] \tag{43}$$

and the updated equation of q_{i1} at $(l+1)^{\text{th}}$ iteration is

$$q_{i1}^{(l+1)} = 1 - \sum_{s=1}^{N} \left[\frac{t_i(z_s, \theta^{(l)})}{\left[\left[\log \frac{a_i}{z_s} - \psi_0(q_{i1}^{(l)} - q_{i2}^{(l)} - 2 + 1) + \psi_0(q_{i1}^{(l)} - 1) \right] t_i(z_s, \theta^{(l)}) \right]} \right] \tag{44}$$

where, $t_i(z_s, \theta^{(l)}) = \left[\frac{\alpha_i^{(l+1)} f_i(z_s, \theta^{(l)})}{\sum_{i=1}^{K} \alpha_i^{(l+1)} f_i(z_s, \theta^{(l)})} \right]$

12. Image Segmentation using Compound Normal with Gamma Mixture Model

In this section, we present the study of image segmentation using compound normal with gamma mixture(CNGM) distribution (Viziananda row, et al.(2015,2016). In this model, the pixel intensities in each image region are assumed to follow Gaussian distribution in which the rate parameter(σ^{-2}) is random and follows a gamma distribution. Compound normal with gamma mixture (CNGM) as given in Norman L. Johnson (2007). The corresponding distribution is defined to have a density function as

$$f(x) = \frac{1}{c^{1/2} B(1/2, \nu/2)} \left[1 + \frac{(x-\mu)^2}{c} \right]^{-(\nu+1)/2} \tag{45}$$

Estimation of the Model Parameters by EM Algorithm

The updated equations are

$$\alpha_l^{(q+1)} = \frac{1}{N}\sum_{i=1}^{N} p^{(q)}\left(l / x_i, \Theta^g\right) \tag{46}$$

$$\mu_l^{(q+1)} = \frac{\sum_{i=1}^{N} x_i p^{(q)}\left(l / x_i, \Theta^g\right)}{N\alpha_l^{(q+1)}} \tag{47}$$

$$v_l^{(q+1)} = \frac{N\alpha_l^{(q+1)}}{\sum_{i=1}^{N}\log\left[1+\frac{\left(x-\mu^{(q+1)}\right)^2}{c_l^{(q)}}\right]p^{(q)}\left(l / x_i, \Theta^g\right)} - 1 \tag{48}$$

$$c_l^{(q+1)} = \frac{\left(v_l^{(q+1)}+1\right)}{N\alpha_l^{(q+1)}}\sum_{i=1}^{N}\left(x_i-\mu_l^{(q+1)}\right)^2 p^q\left(l / x_i, \Theta^g\right) \tag{49}$$

Vizianada Rao et al. (2017) have used doubly truncated compound normal with gamma mixture distribution for segmenting dynamic images where the scale parameter of the distribution of pixel intensities in the image region is a random variable.

13. Image Segmentation Based on Bi-variate Mixture Distributions

Image segmentation is one of the most important areas of image retrieval. In colour image segmentation the feature vector of each image region is '*n*' dimensional different from grey level image. Rajkumar et al. (2011a, 2011b, 2011c) have developed and analyzed image segmentation algorithms using the finite mixture of doubly truncated bi-variate Gaussian distribution. The number of image regions in the whole image is determined using the *K* means or hierarchical clustering algorithms. Assuming that a bi-variate feature vector (consisting of Hue angle and saturation) of each pixel in the image region follows a doubly truncated bi-variate Gaussian distribution, the segmentation algorithm was developed. The model parameters are estimated using EM-Algorithm the updated equations of EM-algorithm for a finite mixture of doubly truncated Gaussian distribution were derived. Segmentation algorithms for colour images were developed by using component maximum likelihood. The performances of the algorithms were evaluated through experimentation with five images taken from Berkeley image dataset and computing the image segmentation metrics such as Global Consistency Error (GCE), Variation of Information (VOI) and Probability Rand Index (PRI). The experimentation results show that this algorithm outperforms the existing other image segmentation algorithms.

Rajkumar et al. (2017) have used doubly truncated bi-variate Gaussian mixture distribution for satellite colour image segmentation to recognize water bodies in deep forest areas. Srinivasa Rao et al. (2012) have developed and analyzed skin color segmentation using finite bi-variate Pearsonian type- IVa mixture model. Jagadesh et al. (2012, 2017) have utilized finite bi-variate Pearsonian type mixture models for skin colour segmentation .Naveen Kumar et al. (2015a, 2015b, 2016a, 2016b) have studied image texture segmentation based on multivariate generalized Gaussian mixture model, under DCT, log DCT, LBP and log DCT+ LBP domains.

14. Experimental Results

In this section we present the performance evaluation of image segmentation method given by Seshashayee et al. (2011a).An experiment was conducted with five images taken from Berkeley image data set

(http://www.eecs.berkeley.edu/Research/Projects/CS/Vision/bsds/BSDS300/html).

The images namely, HORSE, MAN, BIRD, BOAT and TOWER are considered for image segmentation. The pixel intensities of the whole image are taken as feature of the image, and assumed that they follow a generalized new symmetric mixture distribution. In other words, the whole image is collection of *K*-components and the pixel intensities in each component follows a new symmetric distribution. The number of image regions of each image considered for experimentation is determined by hierarchical clustering algorithm.

Using the estimated probability density function and image segmentation algorithm given in section 2, the image segmentation is done for the five images under consideration. The original and segmented images are shown in Figure 2.

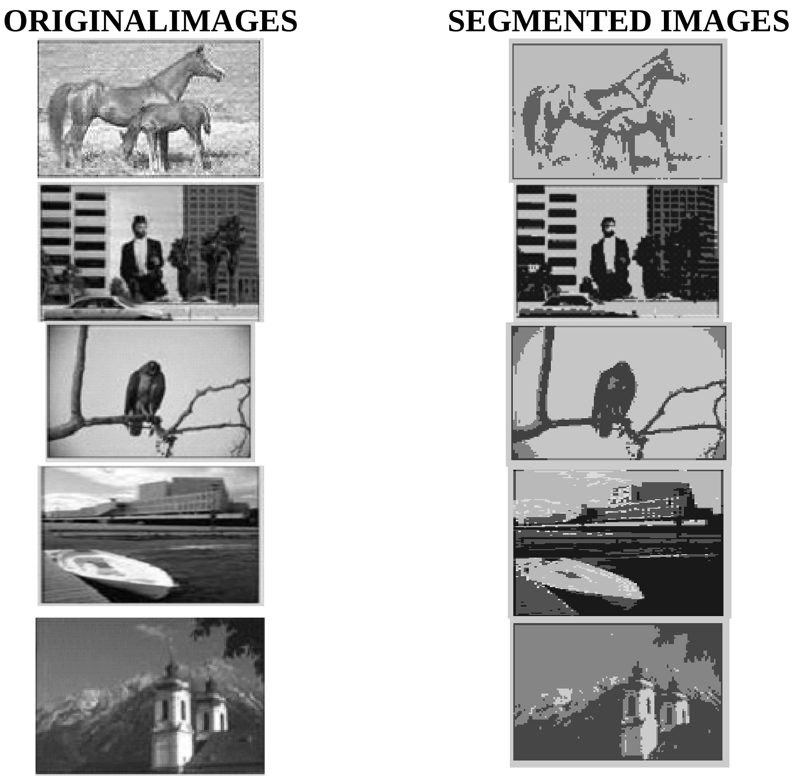

Figure 2: The Original and Segmented Images

Performance Evaluation

After conducting the experiment with the image segmentation algorithm, its performance is studied by obtaining the image segmentation performance measures *viz*., probabilistic rand index (PRI), global consistency error (GCE) and the variation of information (VOI). A comparative study of the developed algorithm with the image segmentation based on generalized new symmetric mixture model with *K*-means. The image segmentation performance measures PRI, VOI, and GCE are computed for all these methods and presented in Table 1.

Table 1: SEGMENTATION PERFORMANCE MEASURES

IMAGE	METHOD	PERFORMANCEMEASURES		
		PRI	GCE	VOI
HORSE	NSMM-K	0.9283	0.1634	1. 8403
	GNSMM-K	0.9374	0.1088	1.8379
	NSMM-H	0.9420	0.1054	1. 8249
	GNSMM-H	0.9596	0.0435	1.7899
MAN	NSMM-K	0.9342	0.1734	1.7875
	GNSMM-K	0.9468	0.1226	1. 7707
	NSMM-H	0.9521	0.0839	1.7366
	GNSMM-H	0.9604	0.0499	1.7254
BIRD	NSMM-K	0.9140	0.1352	1.7259
	GNSMM-K	0.9229	0.1048	1. 6423
	NSMM-H	0.9432	0.0702	1.6373
	GNSMM-H	0.9649	0.0558	1.6321
BOAT	NSMM-K	0.9174	0.6483	1.7542
	GNSMM-K	0.9249	0.2626	1. 7405
	NSMM-H	0.9356	0.1431	1.6980
	GNSMM-H	0.9548	0.1115	1.6587
TOWER	NSMM-K	0.9246	0.0981	1.7988
	GNSMM-K	0.9431	0.0820	1. 7752
	NSMM-H	0.9640	0.0137	1.7539
	GNSMM-H	0.9735	0.0135	1. 7491

From Table 1, it is observed that the segmentation performance measures of the proposed segmentation algorithm are closer to the optimal values of PRI, GCE and VOI.

15. Conclusions

In this paper we have reviewed some of the image segmentation methods based on probability mixture distributions. For gray level images the pixel intensity is considered as the feature, which represents the content of the image more effectively. It is customary to assume that the pixel intensities in an image region are symmetric and mesokurtic and hence, each image region feature is modeled with Gaussian distribution. As a result of it the whole image is characterized by mixture of Gaussian distributions. But in many image regions the pixel intensities may not be distributed as Gaussian since they may be distributed as platykurtic or leptokurtic. In some cases, they may be distributed as asymmetric. It is also to be observed that the tail end probabilities of the pixel intensity distribution cannot be negligible. Therefore, it is needed to consider the image segmentation methods based on non-Gaussian mixture distributions. Accordingly, several authors have developed different image segmentation methods with different mixture distribution to analyze a variety of images. In this article we have discussed image segmentation methods based on mixture of Gaussian distribution, mixture of new symmetric distributions, mixture of generalized new symmetric distributions, mixture of generalized Laplace type distributions, mixture of generalized Gaussian distributions, mixture of doubly truncated Gaussian distributions, mixture of doubly truncated generalized new symmetric distributions, mixture of doubly truncated generalized Gaussian distributions, mixture of Pearsonian type II, mixture of Pearsonian type III, mixture of Pearsonian IVa, mixture of compound normal with Gamma and truncated compound normal with Gamma distributions. The colour image segmentation is developed with a bi-variate feature vector (Hue and Saturation) using doubly truncated bi-variate Gaussian mixture model. The application of image segmentation using probability distributions in remote sensing, skin colour segmentation and texture analysis are pointed. Many more image segmentation methods based on mixture of probability distributions are also available with specific applications. Recently much emphasis is given to estimate model parameter in mixture distributions with EM algorithm. But EM algorithm is sensitive with respect to initial estimates of the parameters. To overcome this problem one can estimate the parameters with Monte Carlo methods of estimation. It is also possible to develop many more image segmentation methods based on mixture of probability distributions.

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