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Some Recursive Constructions of α– Resolvable Group Divisible Designs

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Abstract

Some recursive constructions of α– resolvable group divisible designs with λ_1=0 from certain group divisible designs with λ_1=0 are presented here. In this process some non–isomorphic solutions of group divisible designs are also obtained. A group divisible design with λ_1=0 is used in the construction of Group divisible codes, optimal codes over a cyclic group and LDPC codes, see Ge (2007), Chee *et al.* (2008) and Xu *et al.* (2019). Transversal designs are special classes of such designs.

Keywords: Group divisible designs and codes; Uniform frames; α– resolvable designs; Optimal and LDPC *codes*.

1. Introduction

1.1. Group divisible design

In statistical design theory, a *Group divisible* (*GD*) *design* is defined as an arrangement of v (= mn; m, n ≥2; m groups of n treatments) treatments into b blocks such that each block contains k (<v) distinct treatments, each treatment occurs r times and any pair of distinct treatments which are first associates occur together in λ_1 blocks and in λ_2 blocks if they are second associates. Furthermore, if $r– \lambda_1$=0 then the GD design is singular; if $r– \lambda_1$>0 and $rk–v\lambda_2$=0 then it is semi-regular (*SR*); and if $r– \lambda_1$>0 and $rk–v\lambda_2$>0, the design is regular (*R*).

In Combinatorial design theory; a GD design with index λ is a triple $(\mathcal{V}, \mathcal{G}, \mathcal{B})$ where

(i) $\mathcal{V}$ is a finite set of elements and $\mathcal{G}$ is a set of subsets of $\mathcal{V}$, called groups, which partition $\mathcal{V}$;

(ii) $\mathcal{B}$ is a set of subsets of $\mathcal{V}$, called blocks, such that every pair of elements from distinct groups occurs in exactly λ blocks and $|G \cap B| \leq 1$ for all $G \in \mathcal{G}, B \in \mathcal{B}$.

If all the blocks of the GD design have the same size k and all the groups have the same size n then the GD design is uniform and it is known as (k, λ) – GD design of type n^m for some positive integer m.

Clearly a (k, λ) – GD design of type n^m is a GD design with λ_1=0 (in Statistical design theory). A GD design with λ_1=0 is used in the construction of Group divisible codes, optimal codes over a cyclic group and LDPC codes, see Ge (2007), Chee *et al.* (2008) and Xu *et al.* (2019). A semi- regular GD design with λ_1=0 and $k = m$ is also known as a transversal design in combinatorial design theory.

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1.2. Partial resolution classes and frames

Suppose b blocks of a block design D (v, b, r, k) can be divided into $t = r/\alpha$ classes, each of size $\beta = v\alpha/k$ such that in each class of β blocks every treatment of D is replicated α times. Then these t classes are known as α– resolution (or parallel) classes and the design is called an α– resolvable design. When α=1 the design is said to be resolvable and the classes are called resolution classes.

Let $\mathcal{V}$ be the set of treatments, $\mathcal{G}$ be a set of subsets of $\mathcal{V}$ (called groups), each of size n which partitions $\mathcal{V}$ and $\mathcal{B}$ be the set of subsets of $\mathcal{V}$, called blocks of a GD design. A partial resolution class is a collection of blocks such that every treatment of $\mathcal{V}\backslash G$, $G \in \mathcal{G}$ occurs exactly once and the treatments of G do not occur.

A uniform (k, λ) – frame of index λ is a GD design with parameters v, b, r, k, $\lambda_1 = 0$, $\lambda_2 = \lambda$, m, n such that

(i) the block set $\mathcal{B}$ can be partitioned into a family $\mathcal{R}$ of partial resolution classes, and

(ii) each $R \in \mathcal{R}$ can be associated with a group $G \in \mathcal{G}$ so that R contains every treatment of $\mathcal{V}\backslash G$ exactly once.

Such frame is of type n^m where n is the size of the each group of the GD design. For details see Furino *et al.* (1996; pp. 27– 28) and Ge and Miao (2007).

A comprehensive coverage of constructions of GD designs may be found in Dey (1986, 2010), Raghavarao (1971), Raghavarao and Padgett (2005) and Saurabh *et al.* (2021). Some construction methods of α– resolvable partially balanced incomplete block designs may be found in Sinha and Dey (1982), Kadowaki and Kageyama (2009) and Saurabh and Sinha (2020). Here, some recursive constructions of α– resolvable group divisible designs with λ_1=0 from certain group divisible designs with λ_1=0 are presented. In this process some non–isomorphic solutions of group divisible designs are also obtained. All the group divisible designs constructed here have λ_1=0. *SRX* numbers are from Clatworthy (1973).

2. Recursive Constructions

Theorem 1: The existence of a resolvable SRGD design with parameters

$$v, b = nr, r, k, \lambda_1 = 0, \lambda_2 = \lambda, m, n \tag{1}$$

implies the existence of another resolvable SRGD design with parameters

$$v^* = v + pn, b^* = n^p b, r^* = n^p r, k^* = k + p, \lambda_1^* = 0, \lambda_2^* = n^p \lambda, m^* = m + p, n^* = n \tag{2}$$

p is a positive integer.

Proof: Let $R^1, R^2, \dots, R^r$ be the resolution classes of the SRGD design with parameters (1). Let $B_1^i, B_2^i, \dots, B_n^i$ be arbitrarily chosen blocks in its i^{th} resolution class and $\theta_1, \theta_2, \dots, \theta_n$ be the new treatments other than the v treatments of the GD design. We form b/r resolution classes $R_1^i, R_2^i, \dots, R_{b/r}^i$ corresponding to a resolution class R^i of the SRGD design with parameters (1) as follows:

R_1^i	R_2^i	$\cdots$	$R_{b/r}^i$
$B_1^i \cup \{\theta_1\}$	$B_1^i \cup \{\theta_2\}$	$\cdots$	$B_1^i \cup \{\theta_n\}$
$B_2^i \cup \{\theta_2\}$	$B_2^i \cup \{\theta_1\}$	$\cdots$	$B_2^i \cup \{\theta_{n-1}\}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$B_n^i \cup \{\theta_n\}$	$B_n^i \cup \{\theta_{n-1}\}$	$\cdots$	$B_n^i \cup \{\theta_1\}$

This process is continued for all the resolution classes of the SRGD design. New treatments are added once only in each block. The union of these new resolution classes generates the blocks of another resolvable SRGD design with parameters:

$$v^* = v + n, b^* = nb, r^* = b = nr, k^* = k + 1, \lambda_1^* = 0, \lambda_2^* = \lambda n, m^* = m + 1, n^* = n.$$

Further, by repeated application of this process p (p a positive integer) times we will get a resolvable SRGD design with parameters (2).

Example 1: Consider the following resolution classes of *SR*23: $v = b = 9$, $r = k = 3$, $\lambda_1 = 0$, $\lambda_2 = 1$, $m = n = 3$

R^1	R^2	R^3
1, 2, 3	1, 5, 9	1, 6, 8
4, 5, 6	2, 6, 7	2, 4, 9
7, 8, 9	3, 4, 8	3, 5, 7

Then using Theorem 1, the resolution classes of *SR*43: $v = 12$, $r = 9$, $k = 4$, $b = 27$, $\lambda_1 = 0$, $\lambda_2 = 3$, $m = 4$, $n = 3$ are

R_1^1	R_2^1	R_3^1	R_1^2	R_2^2	R_3^2
1, 2, 3, 10	1, 2, 3, 11	1, 2, 3, 12	1, 5, 9, 10	1, 5, 9, 11	1, 5, 9, 12
4, 5, 6, 11	4, 5, 6, 12	4, 5, 6, 10	2, 6, 7, 11	2, 6, 7, 12	2, 6, 7, 10
7, 8, 9, 12	7, 8, 9, 10	7, 8, 9, 11	3, 4, 8, 12	3, 4, 8, 10	3, 4, 8, 11

R_1^3	R_2^3	R_3^3
1, 6, 8, 10	1, 6, 8, 11	1, 6, 8, 12
2, 4, 9, 11	2, 4, 9, 12	2, 4, 9, 10
3, 5, 7, 12	3, 5, 7, 10	3, 5, 7, 11

Example 2: Consider the following resolution classes of *SR*36: $v = b = 8$, $r = k = 4$, $\lambda_1 = 0$, $\lambda_2 = 2$, $m = 4$, $n = 2$

R^1	R^2	R^3	R^4
1, 2, 3, 4	1, 2, 7, 8	1, 3, 6, 8	1, 4, 6, 7
5, 6, 7, 8	3, 4, 5, 6	2, 4, 5, 7	2, 3, 5, 8

Then using Theorem 1, the resolution classes of *SR*54: $v = 10$, $r = 8$, $k = 5$, $b = 16$, $\lambda_1 = 0$, $\lambda_2 = 4$, $m = 5$, $n = 2$ are

R_1^1	R_2^1	R_1^2	R_2^2	R_1^3	R_2^3
1, 2, 3, 4, 9	1, 2, 3, 4, 10	1, 2, 7, 8, 9	1, 2, 7, 8, 10	1, 3, 6, 8, 9	1, 3, 6, 8, 10
5, 6, 7, 8, 10	5, 6, 7, 8, 9	3, 4, 5, 6, 10	3, 4, 5, 6, 9	2, 4, 5, 7, 10	2, 4, 5, 7, 9

R_1^4	R_2^4
1, 4, 6, 7, 9	1, 4, 6, 7, 10
2, 3, 5, 8, 10	2, 3, 5, 8, 9

Theorem 2: The existence of an n– resolvable SRGD design with parameters (1) implies the existence of another n- resolvable SRGD design with parameters

$$v^* = v + pn, b^* = n^p b, r^* = n^p r, k^* = k + p, \lambda_1^* = 0, \lambda_2^* = n^p \lambda, m^* = m + p, n^* = n. \quad (3)$$

p is a positive integer.

Proof: Since the SRGD design with parameters (1) is n-resolvable, the number of resolution classes is r/n and the number of blocks in each resolution class is $bn/r = vn/k = n^2$. Let $R^1, R^2, \dots, R^{r/n}$ be the resolution classes of SRGD design with parameters (1). Let $B_1^i, B_2^i, \dots, B_{n^2}^i$ be arbitrarily chosen blocks in its i^{th} resolution class and $\theta_1, \theta_2, \dots, \theta_n$ be the new treatments distinct from the v treatments of the SRGD design. We construct n resolution classes corresponding to a resolution class R^i of the SRGD design as follows:

R_1^i	R_2^i	…	R_n^i
$B_1^i \cup \{\theta_1\}$	$B_1^i \cup \{\theta_2\}$	$\cdots$	$B_1^i \cup \{\theta_n\}$
$B_2^i \cup \{\theta_1\}$	$B_2^i \cup \{\theta_2\}$	$\cdots$	$B_2^i \cup \{\theta_n\}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$B_n^i \cup \{\theta_1\}$	$B_n^i \cup \{\theta_2\}$	$\cdots$	$B_n^i \cup \{\theta_n\}$
$B_{n+1}^i \cup \{\theta_2\}$	$B_{n+1}^i \cup \{\theta_1\}$	$\cdots$	$B_{n+1}^i \cup \{\theta_{n-1}\}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$B_{2n}^i \cup \{\theta_2\}$	$B_{2n}^i \cup \{\theta_1\}$	$\cdots$	$B_{2n}^i \cup \{\theta_{n-1}\}$
$\vdots$	$\vdots$	$\cdots$	$\vdots$
$B_{n^2-1}^i \cup \{\theta_n\}$	$B_{n^2-1}^i \cup \{\theta_{n-1}\}$	$\cdots$	$B_{n^2-1}^i \cup \{\theta_1\}$
$B_{n^2}^i \cup \{\theta_n\}$	$B_{n^2}^i \cup \{\theta_{n-1}\}$	$\cdots$	$B_{n^2}^i \cup \{\theta_1\}$

We continue this process for all the resolution classes of the SRGD design with parameters (1). New treatments are added once only in each block. The union of these new resolution classes generates the blocks of another n-resolvable SRGD design with parameters:

$$v^* = v + n, b^* = bn, r^* = b = nr, k^* = k + 1, \lambda_1^* = 0, \lambda_2^* = n\lambda, m^* = m + 1, n^* = n.$$

Further, by repeated application of this process p (p a positive integer) times we will get an n-resolvable SRGD design with parameters (3).

Example 3: Consider the following 2– resolvable solution of *SR*66: $v = 12$, $b = 8$, $r = 4$, $k = 6$, $\lambda_1 = 0$, $\lambda_2 = 2$, $m = 6$, $n = 2$:

R^1	R^2
1, 2, 3, 4, 5, 6	1, 3, 8, 5, 10, 12
5, 6, 7, 8, 9, 10	2, 4, 5, 7, 9, 12
1, 2, 9, 10, 11, 12	1, 4, 6, 8, 9, 11
3, 4, 7, 8, 11, 12	2, 3, 6, 7, 10, 11

Then using Theorem 2, a 2– resolvable solution of *SR*82: $v = 14$, $r = 8$, $k = 7$, $b = 16$, $\lambda_1 = 0$, $\lambda_2 = 4$, $m = 7$, $n = 2$ is obtained as:

R_1^1	R_2^1	R_1^2	R_2^2
1, 2, 3, 4, 5, 6, 13	1, 2, 3, 4, 5, 6, 14	1, 3, 8, 5, 10, 12, 14	1, 3, 8, 5, 10, 12, 13
5, 6, 7, 8, 9, 10, 13	5, 6, 7, 8, 9, 10, 14	2, 4, 5, 7, 9, 12, 14	2, 4, 5, 7, 9, 12, 13
1, 2, 9, 10, 11, 12, 14	1, 2, 9, 10, 11, 12, 13	1, 4, 6, 8, 9, 11, 13	1, 4, 6, 8, 9, 11, 14
3, 4, 7, 8, 11, 12, 14	3, 4, 7, 8, 11, 12, 13	2, 3, 6, 7, 10, 11, 13	2, 3, 6, 7, 10, 11, 14

Remark 1: Clatworthy (1973) reported a resolvable and a 4-resolvable solution for *SR*82 while the solution presented here is 2-resolvable. Hence the present solution is non– isomorphic.

The following Table lists *n*-resolvable ($n \geq 1$) solutions of some SRGD designs using Theorems 1 and 2 with $p = 1$:

Table 1: SRGD Designs

No.	Original design	Derived design	Source
1	*SR*1, Resolvable	*SR*19, Resolvable	Th. 1
2	*SR*6, Resolvable	*SR*25, Resolvable	Th. 1
3	*SR*23, Resolvable	*SR*43, Resolvable	Th. 1
4	*SR*36, Resolvable and 2– resolvable	*SR*54, Resolvable and 2– resolvable	Th. 1 & 2
5	*SR*52, 2– resolvable	*SR*69, 2– resolvable	Th. 2
6	*SR*66, 2– resolvable	*SR*82, 2– resolvable, Non- isomorphic	Th. 2

Theorem 3: The existence of a uniform (k, λ) – frame with parameters: $v = b$, $r = k$, $\lambda_1 = 0$, $\lambda_2 = \lambda$, m, n implies the existence of a resolvable GD design with parameters

$$v^* = v, b^* = nv, r^* = n(k+1), k^* = k+1, \lambda_1^* = 0, \lambda_2^* = \lambda n\,(k+1)/(k-1)\,, m^* = m,$$
$$n^* = n. \tag{4}$$

where n is equal to the number of blocks in partial resolution class of a uniform (k, λ) – frame.

Proof: Let $R^1, R^2, \ldots, R^t$ be the partial resolution classes. Let $B_1^i, B_2^i, \ldots, B_n^i$ be arbitrarily chosen blocks and $\{\theta_1, \theta_2, \ldots, \theta_n\}$ be the missing group in i^{th} partial resolution class of the uniform (k, λ) – frame. We form n resolution classes corresponding to a partial resolution class R^i as follows:

R_1^i	R_2^i	…	R_n^i
$B_1^i \cup \{\theta_1\}$	$B_1^i \cup \{\theta_2\}$	$\cdots$	$B_1^i \cup \{\theta_n\}$
$B_2^i \cup \{\theta_2\}$	$B_2^i \cup \{\theta_1\}$	$\cdots$	$B_2^i \cup \{\theta_{n-1}\}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$B_n^i \cup \{\theta_n\}$	$B_n^i \cup \{\theta_{n-1}\}$	$\cdots$	$B_n^i \cup \{\theta_1\}$

We continue this process for all the partial resolution classes of an (k, λ) – frame. One of the n treatments from missing groups are added once only in each block. The union of these new resolution classes generates the blocks of a resolvable GD design with parameters (4).

Example 4: Consider a (3, 1) – frame of type 2^4 whose partial resolution classes are:

Partial Resolution Classes	R^1	R^2	R^3	R^4
groups	{1, 5}	{2, 4}	{3, 6}	{7, 8}
blocks	{2, 6, 7} {3, 4, 8}	{1, 6, 8} {3, 5, 7}	{1, 4, 7} {2, 5, 8}	{1, 2, 3} {4, 5, 6}

Then using Theorem 3, we obtain a resolvable GD design with parameters *SR*39: $v = 8$, $r = 8$, $k = 4$, $b = 16$, $\lambda_1 = 0$, $\lambda_2 = 4$, $m = 4$, $n = 2$ whose resolution classes are:

R_1^1	R_2^1	R_1^2	R_2^2	R_1^3	R_2^3
{1, 2, 6, 7} {3, 4, 5, 8}	{2, 5, 6, 7} {1, 3, 4, 8}	{1, 2, 6, 8} {3, 4, 5, 7}	{1, 4, 6, 8} {2, 3, 5, 7}	{1, 3, 4, 7} {2, 5, 6, 8}	{1, 4, 6, 7} {2, 3, 5, 8}

R_1^4	R_2^4
{1, 2, 3, 7} {4, 5, 6, 8}	{1, 2, 3, 8} {4, 5, 6, 7}

Theorem 4: The existence of a nonresolvable SRGD design with parameters (1) implies the existence of another *r*–resolvable SRGD design with parameters

$$v^* = v + n, b^* = n^2 r, r^* = nr, k^* = k + 1, \lambda_1^* = 0, \lambda_2^* = \lambda n, m^* = m + 1, n^* = n. \qquad (5)$$

Proof: Let $B_1^i, B_2^i, \dots, B_{nr}^i$ be arbitrarily chosen blocks of the nonresolvable SRGD design with parameters (1) and $\theta_1, \theta_2, \dots, \theta_n$ be the new treatments other than *v* treatments of the SRGD design. We constitute an *r*– resolvable solution of a GD design with parameters (5) whose blocks are given as follows:

R_1	R_2	...	R_n
$B_1 \cup \{\theta_1\}$	$B_1 \cup \{\theta_2\}$	$\cdots$	$B_1 \cup \{\theta_n\}$
$B_2 \cup \{\theta_1\}$	$B_2 \cup \{\theta_2\}$		$B_2 \cup \{\theta_n\}$
$\vdots$	$\vdots$	$\cdots$	$\vdots$
$B_r \cup \{\theta_1\}$	$B_r \cup \{\theta_2\}$		$B_r \cup \{\theta_n\}$
$B_{r+1} \cup \{\theta_2\}$	$B_{r+1} \cup \{\theta_1\}$	$\cdots$	$B_{r+1} \cup \{\theta_{n-1}\}$
$\vdots$	$\vdots$		$\vdots$
$B_{2r} \cup \{\theta_2\}$	$B_{2r} \cup \{\theta_1\}$	$\cdots$	$B_{2r} \cup \{\theta_{n-1}\}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$B_{nr-1} \cup \{\theta_n\}$	$B_{nr-1} \cup \{\theta_{n-1}\}$	$\cdots$	$B_{nr-1} \cup \{\theta_1\}$
$B_{nr} \cup \{\theta_n\}$	$B_{nr} \cup \{\theta_{n-1}\}$	$\cdots$	$B_{nr} \cup \{\theta_1\}$

New treatments are added once only in each block.

When *r*=*n* in Theorem 4, by the repeated application of the process in Theorem 2 we get:

Corollary 1: The existence of a nonresolvable SRGD design with parameters (1) implies the existence of another *n*– resolvable SRGD design with parameters:

$$v^* = v + (p+1)n, b^* = n^{p+3}, r^* = n^{p+2}, k^* = k + p + 1, \lambda_1^* = 0, \lambda_2^* = \lambda n^{p+1}, \ m^* = m + p + 1, n^* = n; p \text{ is a positive integer.} \qquad (6)$$

Example 5: Consider a SRGD design *SR*41: $v = 12$, $b = 9$, $r = 3$, $k = 4$, $\lambda_1 = 0$, $\lambda_2 = 1$, $m = 4$, $n = 3$ whose blocks are given as:

{1, 2, 3, 4}, {4, 5, 7, 10}, {4, 6, 9, 11}, {1, 6, 7, 8}, {2, 5, 8, 11}, {3, 8, 9, 10}, {1, 10, 11, 12}, {2, 7, 9, 12}, {3, 5, 6, 12}.

Then using Theorem 4, a 3-resolvable solution of *SR*57: $v = 15$, $b = 27$, $r = 9$, $k = 5$, $\lambda_1 = 0$, $\lambda_2 = 3$, $m = 5$, $n = 3$ is given as:

R_1	R_2	R_3
{1, 2, 3, 4, 13}	{1, 2, 3, 4, 14}	{1, 2, 3, 4, 15}
{4, 5, 7, 10, 13}	{4, 5, 7, 10, 14}	{4, 5, 7, 10, 15}
{4, 6, 9, 11, 13}	{4, 6, 9, 11, 14}	{4, 6, 9, 11, 15}
{1, 6, 7, 8, 14}	{1, 6, 7, 8, 15}	{1, 6, 7, 8, 13}
{2, 5, 8, 11, 14}	{2, 5, 8, 11, 15}	{2, 5, 8, 11, 13}
{3, 8, 9, 10, 14}	{3, 8, 9, 10, 15}	{3, 8, 9, 10, 13}
{1, 10, 11, 12, 15}	{1, 10, 11, 12, 13}	{1, 10, 11, 12, 14}
{2, 7, 9, 12, 15}	{2, 7, 9, 12, 13}	{2, 7, 9, 12, 14}
{3, 5, 6, 12, 15}	{3, 5, 6, 12, 13}	{3, 5, 6, 12, 14}

Since $r = n = 3$ here, following Corollary 1 for $p = 1$ we will get a SRGD design with parameters: $v = 18$, $b = 81$, $r = 27$, $k = 6$, $\lambda_1 = 0$, $\lambda_2 = 9$, $m = 6$, $n = 3$.

Remark 2: Clatworthy (1973) reported a resolvable solution for *SR*57 whereas a 3-resolvable solution is obtained here for the same. Hence the present solution is non– isomorphic.

Remark 3: The association scheme of the derived GD design in the Theorems 1, 2 and 4 is obtained by adjoining a new row: $mn+1$, $mn+2,\ldots, n(m+1)$ to the $m\times n$ association scheme of the original GD design.

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